Soundly Handling Linearity

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Seminar, University of Bristol, 3rd Oct 2023

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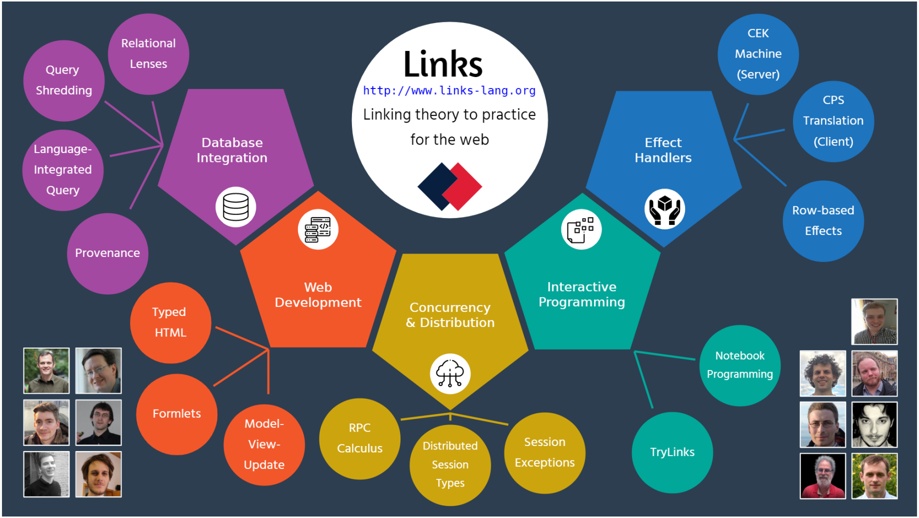
Links



Picture by Simon Fowler

Linear Types in Links

Linear types statically guarantee linear resources are used exactly once.

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LINKS uses linear types for session types:

- `!A.S` : send a value of type `A`, then continue as `S`
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Primitive operations on session-typed channels:

```
send    : forall (a::Any) (b::Session) . (a, !a.b) -> b
receive : forall (a::Any) (b::Session) . (?a.b) -> (a, b)
fork    : forall          (b::Session) . (b -> ()) -> ~b
close   : End -> ()
```

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A sender sends an integer.

```
sig sender      : (!Int.End) ~> ()
fun sender(c)   { var c' = send(42, c); close(c') }
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Fork the receiver and pass the dual channel to the sender.

```
links> { var c = fork(receiver); sender(c) };
42
```

Linear types in LINKS are sound

Linear channels cannot be used twice.

```
links> { var c = fork(receiver); sender(c); sender(c); };
Type error: Variable ch has linear type `!Int.End' but is used 2 times.
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Unlimited functions cannot capture linear channels.

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links> { var c = fork(receiver);
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fun choose() { var i = if (do Choose) 42 else 84; printInt(i) }
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links> handle (choose())
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4284
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Well-typed programs in LINKS can go wrong ! [1][2]

A nondeterministic sender sends an integer using the `Choose` operation.

```
sig ndsender : forall r::Row . (!Int.End) { Choose: () => Bool | r}~> ()
fun ndsender(c) {var c' = send(if (do Choose) 42 else 84, c); close(c')}
```

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Use the same channel twice by multi-shot handlers.

```
links> handle ({ var c = fork(receiver); ndsender(c) })
       { case <Choose => r> -> r(true); r(false) };
```

```
42***: Internal Error in evalir.ml (Please report as a bug):
NotFound chan_3 (in Hashtbl.find) while interpreting.
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Our solution: track *control-flow linearity* in addition to value linearity.

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Value Linearity in F°_{eff}

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F°_{eff} tracks the value linearity with kinds.

$$
\begin{aligned}
\mathit{Int} &: \mathsf{Type}^{\bullet} \\
\mathit{File} &: \mathsf{Type}^{\circ} \\
(\mathit{File}, \mathit{Int}) &: \mathsf{Type}^{\circ} \\
A \to^{\circ} C &: \mathsf{Type}^{\circ}
\end{aligned}
$$

Value Linearity in $\mathsf{F}^{\circ}_{\text{eff}}$

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&A \to^{\circ} C && : \mathsf{Type}^{\circ}
\end{aligned}$$

Functions are annotated with their value linearity.

$$\begin{aligned}
&\mathit{faithfulWrite} \ : \ \mathit{File} \to^{\bullet} (\mathit{String} \to^{\circ} ()) \\
&\mathit{faithfulWrite} = \lambda^{\bullet} f.(\lambda^{\circ} s.\mathbf{let} \ f' \leftarrow \mathit{write}\,(s, f) \ \mathbf{in} \ \mathit{close}\, f')
\end{aligned}$$

Unlimited values can be used as linear values

It is always safe to use unlimited values just once.

$$\begin{aligned} id\ &: \alpha^{\mathsf{Type}^\circ}.\, \alpha \to^\bullet \alpha \,!\, \{\} \\ id &= \alpha^{\mathsf{Type}^\circ}.\, \lambda^\bullet x.\, x \end{aligned}$$

With the *subkinding* relation $\vdash \mathbf{Type}^\bullet \leq \mathbf{Type}^\circ$, we can instantiate α to *Int*.

$$\begin{aligned} id\ File &: File \to^\bullet File \,!\, \{\} \\ id\ Int &: Int \to^\bullet Int \,!\, \{\} \end{aligned}$$

Multi-shot handlers abuse linear resources

We encounter the same problem as LINKS if we only track value linearity in the presence of multi-shot handlers.

$$\begin{aligned}
&\mathit{dubiousWrite}_{\text{✗}} \ : \ \mathit{File} \to^{\bullet} () \,!\, \{\mathit{Choose} : () \twoheadrightarrow \mathit{Bool}\} \\
&\mathit{dubiousWrite}_{\text{✗}} = \lambda^{\bullet} f. \\
&\quad \textbf{let } b \leftarrow (\textbf{do } \mathit{Choose}\, ())^{\{\mathit{Choose}:() \twoheadrightarrow \mathit{Bool}\}} \textbf{ in} \\
&\quad \left.\begin{array}{l} \textbf{let } s \leftarrow \textbf{if } b \textbf{ then } \text{"}A\text{"} \textbf{ else } \text{"}B\text{"} \textbf{ in} \\ \textbf{let } f' \leftarrow \mathit{write}\,(s, f) \textbf{ in } \mathit{close}\, f' \end{array}\right\} \text{continuation of } \mathit{Choose}
\end{aligned}$$

$$\begin{aligned}
&\textbf{let } f \leftarrow \mathit{open}\, \text{"C.txt"} \textbf{ in} \\
&\textbf{handle } (\mathit{dubiousWrite}_{\text{✗}}\ f) \textbf{ with } \{\mathit{Choose}\ _\ r \mapsto r\ \mathit{true}\,; r\ \mathit{false}\}
\end{aligned}$$

Control-Flow Linearity in F°_{eff}

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The continuation (context) of *Choose* is control-flow linear.

$$
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\end{aligned}
$$

Control-Flow Linearity in $\mathrm{F}^{\circ}_{\mathrm{eff}}$

Linearity $Y ::= \circ \mid \bullet$

$\mathrm{F}^{\circ}_{\mathrm{eff}}$ tracks CFL at the granularity of operations ($\mathit{Choose} : () \twoheadrightarrow^{Y} \mathit{Bool}$), which represents the CFL of their continuations.

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Let-bindings ($\mathbf{let}^{Y}\ x \leftarrow M\ \mathbf{in}\ N$) are annotated with the CFL of the local context of M (i.e., $\mathbf{let}^{Y}\ x \leftarrow \_\ \mathbf{in}\ N$).

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$$
\begin{aligned}
&\mathit{dubiousWrite}_{\checkmark} \;:\; \mathit{File} \to^{\bullet} ()\,!\,\{\mathit{Choose} : () \twoheadrightarrow^{\circ} \mathit{Bool}\}\\
&\mathit{dubiousWrite}_{\checkmark} = \lambda^{\bullet} f.\\
&\quad \mathbf{let}^{\circ} b \leftarrow (\mathbf{do}\; \mathit{Choose}\,())^{\{\mathit{Choose}:() \twoheadrightarrow^{\circ} \mathit{Bool}\}} \;\mathbf{in}\\
&\quad \left.\begin{aligned}&\mathbf{let}^{\circ} s \leftarrow \mathbf{if}\; b \;\mathbf{then}\; \text{"A"} \;\mathbf{else}\; \text{"B"} \;\mathbf{in}\\ &\mathbf{let}^{\bullet} f' \leftarrow \mathit{write}\,(s, f) \;\mathbf{in}\; \mathit{close}\, f'\end{aligned}\right\}\text{continuation of } \mathit{Choose}
\end{aligned}
$$

$$
\begin{aligned}
&\mathbf{let}\; f \leftarrow \mathit{open}\, \text{"C.txt"} \;\mathbf{in}\\
&\mathbf{handle}\; (\mathit{dubiousWrite}_{\checkmark}\; f) \;\mathbf{with}\; \{\mathit{Choose}\; \_\; r \mapsto r\; \mathit{true}\,; r\; \mathit{false}\}
\end{aligned}
$$

Ill-typed as r is given a linear function type!

Linear effect rows can be used as unlimited ones

F°_{eff} lifts the control-flow linearity of operations to effect rows.

$$(\mathit{Choose} : () \twoheadrightarrow^{\circ} \mathit{Bool}) \quad : \mathrm{Row}^{\circ}$$
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$$(L_1 : \circ ; L_2 : \circ ; L_3 : \bullet) \quad : \mathrm{Row}^{\bullet}$$

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\end{aligned}$$

It is always safe to use control-flow-linear operations in an unlimited context.

$$\begin{aligned}
&\mathit{tossCoin} \ : \ \forall \mu^{\mathrm{Row}^{\bullet}}.(() \to^{\bullet} \mathit{Bool}\,!\,\{\mu\}) \to^{\bullet} \mathit{String}\,!\,\{\mu\} \\
&\mathit{tossCoin} = \Lambda \mu^{\mathrm{Row}^{\bullet}}.\lambda^{\bullet} g.\ \mathbf{let}^{\bullet}\ b \leftarrow g\ ()\ \mathbf{in}\ \mathbf{if}\ b\ \mathbf{then}\ \text{"}\mathit{heads}\text{"}\ \mathbf{else}\ \text{"}\mathit{tails}\text{"}
\end{aligned}$$

With the *subkinding* relation $\vdash \mathrm{Row}^{\circ} \leq \mathrm{Row}^{\bullet}$, we have

$$\begin{aligned}
&\mathit{tossCoin}\ \{\mathit{Choose} : \bullet\}\ (\lambda^{\bullet}().(\mathbf{do}\ \ \mathit{Choose}\,())^{\{\mathit{Choose}:\bullet\}}) \\
&\mathit{tossCoin}\ \{\mathit{Choose} : \circ\}\ (\lambda^{\bullet}().(\mathbf{do}\ \ \mathit{Choose}\,())^{\{\mathit{Choose}:\circ\}})
\end{aligned}$$

Control flow linearity is *“dual”* to value linearity!

Control-Flow Linearity in Links

Previously, Links does not track control-flow linearity.

```
links> fun(ch:End) {do L; close(ch)};
fun : (End) {L:() => () | _}~> ()
```

Control-Flow Linearity in LINKS

Previously, LINKS does not track control-flow linearity.

```
links> fun(ch:End) {do L; close(ch)};
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By default, CFL is unlimited. We use the keyword **`xlin`** to switch CFL to linear, and **`lindo`** to invoke control-flow-linear operations.

```
links> fun(ch:End) {xlin; lindo L; close(ch)};
fun : () {L:() =@ () | _::Lin}~> ()
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links> fun(ch:End) {xlin; lindo L; close(ch)};
fun : () {L:() =@ () | _::Lin}~> ()
```

Control-flow-linear operations can only be handled by one-shot handlers.

```
links> fun(ch:End) { handle ({xlin; lindo L; close(ch)})
                     {case <L =@ r> -> xlin; r(())} };
fun : (End) {L{_::Lin}|_::Lin}~> ()
```

Nondeterministic sender, again

```
sig receiver : (?Int.End) { |_::Lin}~> ()
fun receiver(c) { xlin; var (i, c') = receive(c); close(c'); printInt(i) }
sig ndsender : (!Int.End) {Choose: () => Bool | _::Lin}~> ()
fun ndsender(c) {xlin; close(send(if (lindo Choose) 42 else 84, c))}
```

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```
links> handle ({ xlin; var c = fork(receiver); ndsender(c) })
       { case <Choose => r> -> r(true); r(false) };
  Type error: ... =@ does not match => ...
```

```
links> handle ({ xlin; var c = fork(receiver); ndsender(c) })
       { case <Choose =@ r> -> r(true); r(false) };
  Type error: ... linear function r is used 2 times ...
```

```
links> handle ({ xlin; var c = fork(receiver); ndsender(c) })
       { case <Choose =@ r> -> r(true) };
42
```

Implementation Details

LINKS also adapts a Row-based effect system. Effect types of sequenced computations are unified. For instance,

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f(42); g(); h("Hello, world!")
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Informally, we introduce the concept *effect scope* to mean the maximal scope where computations have the same effect types. There are only two cases that new effect scopes are created:

- Function bodies (closures) hold their own effect scopes.
- Computations being handled (the `M` in **`handle`** `M {...}`) have their own effect scopes, but also share unhandled effects with outside.

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`xlin` requires all operations in the current effect scope to be linear.

(Bonus) xlin is a modality ?

Intuition: **`xlin`** creates a linear scope.

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Typing rules for the Fitch-style modal lambda calculus λ_{IK}:

$$\frac{✂ \notin \Gamma'}{\Gamma, x : A, \Gamma' \vdash x : A}$$

$$\frac{\Gamma, ✂ \vdash M : A}{\Gamma \vdash \mathbf{box}\, M : \Box A}$$

$$\frac{\Gamma \vdash M : \Box A \qquad ✂ \notin \Gamma'}{\Gamma, ✂, \Gamma' \vdash \mathbf{unbox}\, M : A}$$

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TLDR: No, it isn't.

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If we only consider where linear variables can be used

$$\text{T-VAR}\;\frac{✂ \notin \Gamma'}{\Gamma, x : A, \Gamma' \vdash x : A}
\qquad
\text{T-BOX(CT)}\;\frac{\Gamma, [✂] \vdash V : A}{\Gamma \vdash \mathbf{box}\, V : \Box A}
\qquad
\text{T-UNBOX(4)}\;\frac{\Gamma \vdash V : \Box A}{\Gamma, ✂, \Gamma' \vdash \mathbf{unbox}\, V : A}$$

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If we only consider where linear variables can be used

T-Var
$$\frac{✂ \notin \Gamma'}{\Gamma, x : A, \Gamma' \vdash x : A}$$

T-Box(CT)
$$\frac{\Gamma, [✂] \vdash V : A}{\Gamma \vdash \mathbf{box}\ V : \square A}$$

T-Unbox(4)
$$\frac{\Gamma \vdash V : \square A}{\Gamma, ✂, \Gamma' \vdash \mathbf{unbox}\ V : A}$$

However, it doesn't work well for operations :(

The main problem is that closures should create new scopes.

(Bonus) CFL with modalities

We may still formalise **`xlin`** with modalities.

(Bonus) CFL with modalities

We may still formalise **`xlin`** with modalities.

Consider CBPV. Value linearity is a property of values, while CFL is a property of computations (effects). $\Box A$ and $\Box E$ for unlimited values and effects.

$$\frac{✂ \notin \Gamma'}{\Gamma, x : A, \Gamma' \vdash x : A} \qquad \frac{\Gamma, ✂ \vdash V : A}{\Gamma \vdash \mathbf{box}\, V : \Box A} \qquad \frac{\Gamma \vdash V : \Box A}{\Gamma, \Gamma' \vdash \mathbf{unbox}\, V : A}$$

$$\frac{\Gamma \vdash M : C \dashv E}{\Gamma \vdash \mathbf{thunk}\, M : {\downarrow^E} C} \qquad \frac{\Gamma \vdash V : {\downarrow^E} C}{\Gamma \vdash \mathbf{force}\, V : C \dashv E} \qquad \frac{\Gamma \vdash M : C \dashv \Box E \qquad ✂ \notin \Gamma'}{\Gamma, ✂, \Gamma' \vdash \mathbf{unbox}\, M : C \dashv E}$$

$$\frac{\Gamma \vdash M : {\uparrow} A \dashv E_1 \qquad \Gamma, ✂, x : A \vdash N : C \dashv E_2}{\Gamma \vdash \mathbf{let\ box}\ x \leftarrow M\ \mathbf{in}\ N : C \dashv (\Box E_1) \cup E_2}$$

$$\frac{\Gamma \vdash M : {\uparrow} A \dashv E_1 \qquad \Gamma, x : A \vdash N : C \dashv E_2}{\Gamma \vdash \mathbf{let}\ x \leftarrow M\ \mathbf{in}\ N : C \dashv (\mathsf{lin}(E_1)) \cup E_2}$$

Restriction of Subkinding-based Linear Types

Linear types in $\mathsf{F}^{\circ}_{\text{eff}}$ (and LINKS) can be annoying due to annotations and lack of principal types.

$$\mathit{verboseId} \;:\; \forall \mu^{\mathrm{Row}^{Y_1}}\ \alpha^{\mathrm{Type}^{Y_2}}.\, \alpha \longrightarrow^{Y_0} \alpha\,!\,\{\mathit{Print} : \mathit{String} \twoheadrightarrow^{Y_3} ()\,;\mu\}$$
$$\mathit{verboseId} = \Lambda \mu^{\mathrm{Row}^{Y_1}}\ \alpha^{\mathrm{Type}^{Y_2}}.\, \lambda^{Y_0} x.\, \mathbf{let}^{Y_4}\ () \leftarrow \mathbf{do}\ \mathit{Print}\ \text{"}\mathit{idiscalled}\text{"}\ \mathbf{in}\ x$$

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We have ten different types for *verboseId*, none of which is the most general.

$$\forall \mu^{\bullet}\ \alpha^{\bullet}.\alpha \to^{\bullet} \alpha\,!\,\{\mathit{Print} : \bullet\,;\mu\} \qquad \forall \mu^{\bullet}\ \alpha^{\bullet}.\alpha \to^{\circ} \alpha\,!\,\{\mathit{Print} : \bullet\,;\mu\}$$
$$\forall \mu^{\bullet}\ \alpha^{\bullet}.\alpha \to^{\bullet} \alpha\,!\,\{\mathit{Print} : \circ\,;\mu\} \qquad \forall \mu^{\bullet}\ \alpha^{\bullet}.\alpha \to^{\circ} \alpha\,!\,\{\mathit{Print} : \circ\,;\mu\}$$
$$\forall \mu^{\circ}\ \alpha^{\bullet}.\alpha \to^{\bullet} \alpha\,!\,\{\mathit{Print} : \bullet\,;\mu\} \qquad \forall \mu^{\circ}\ \alpha^{\bullet}.\alpha \to^{\circ} \alpha\,!\,\{\mathit{Print} : \bullet\,;\mu\}$$
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$$\forall \mu^{\circ}\ \alpha^{\circ}.\alpha \to^{\bullet} \alpha\,!\,\{\mathit{Print} : \circ\,;\mu\} \qquad \forall \mu^{\circ}\ \alpha^{\circ}.\alpha \to^{\circ} \alpha\,!\,\{\mathit{Print} : \circ\,;\mu\}$$

Qualified Linear Types in $\mathrm{Q}^{\circ}_{\mathrm{eff}}$

We can restore principal types by abstracting over linearity and introducing constraints on linearity.

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The order of linearity is given by $\bullet \leq \circ$.

$\alpha \leq \phi$: the linearity of the value type α is less than the linearity variable ϕ

$\alpha \leq \mu$: the linearity of the value type α is less than the control-flow linearity of the row type μ

Restriction of Row-based Effect Types

Effect row types of sequenced computations must be unified.

$$\mathit{sandwichClose} \ : \ (() \to^{\bullet} () \,!\, \{R_1\}, \mathit{File}, () \to^{\bullet} () \,!\, \{R_2\}) \to^{\bullet} () \,!\, \{R\}$$
$$\mathit{sandwichClose} = \lambda^{\bullet}(g, f, h).\ \mathbf{let}^{\circ}\,() \leftarrow g\,()\ \mathbf{in}\ \mathbf{let}^{\bullet}\,() \leftarrow \mathit{close}\,f\ \mathbf{in}\ h\,()$$

We can only have $R_1 = R_2 = R$, which overly restricts that operations invoked in h must be control-flow linear.

Qualified Effect Types in Q°_{eff}

We support row subtyping again by qualified types.

$$\begin{aligned} \mathit{sandwichClose} \;&:\; \forall \mu_1\, \mu_2\, \mu.(\mu_1 \leqslant \mu, \mu_2 \leqslant \mu, \mathit{File} \preceq \mu_1) \\ &\quad\Rightarrow ((() \to^{\bullet} ()\,!\,\{\mu_1\}, \mathit{File}, () \to^{\bullet} ()\,!\,\{\mu_2\}) \to^{\bullet} ()\,!\,\{\mu\} \\ \mathit{sandwichClose} \;&=\; \lambda^{\bullet}(g, f, h).\ \mathbf{let}\ () \leftarrow g\,()\ \mathbf{in}\ \mathbf{let}\ () \leftarrow \mathit{close}\, f\ \mathbf{in}\ h\,() \end{aligned}$$

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But having explicit constraint sets in types is still a pain?

(Bonus) Algebraic Subtyping for Effects

Use algebraic subtyping.

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The core idea of algebraic subtyping is to encode subtyping constraints with union and intersection directly in types. For instance,

$$\forall \alpha\, \beta\, \gamma.(\alpha \leqslant \gamma, \beta \leqslant \gamma) \Rightarrow (\alpha, \beta) \to \gamma$$

is transformed to

$$\forall \alpha\, \beta.(\alpha, \beta) \to \alpha \sqcup \beta$$

Algebraic subtyping for row types is quite standard. Informally,

$$\frac{\Gamma \vdash M : A \,!\, R_1 \qquad N : B \,!\, R_2}{\Gamma \vdash M; N : B \,!\, R_1 \sqcup R_2}$$

$R_1 \sqcup R_2$: the union of row types R_1 and R_2

(Bonus) Algebraic Subtyping for Linearity

Algebraic subtyping for linear types is more interesting. Informally,

$$\begin{aligned} &\lambda x.\lambda y.\lambda z.(x, y, z) &&: \alpha \to \beta \to^{\alpha} \gamma \to^{\alpha \vee \beta} (\alpha, \beta, \gamma) \\ &\lambda x.(x, x) &&: \alpha \wedge \bullet \to (\alpha, \alpha) \end{aligned}$$

$\to^{\alpha}$: a function type whose linearity is *at least* the linearity of α

$\alpha \vee \beta$: the union of the linearity of value types α and β

$\alpha \wedge \bullet$: α with linearity that is the intersection of α and $\bullet$

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It is easy to extend it with control flow linearity. Informally,

$$\begin{aligned} \mathit{verboseId} &: \alpha \to \alpha \,!\, \{\mathit{Print} : \phi \vee \alpha \,;\, \mu\} \\ \mathit{verboseId} &= \lambda x.\, \mathbf{do}\ \mathit{Print}\ \text{"idiscalled"}\,;\, x \end{aligned}$$

Conclusion

More in the paper: `https://arxiv.org/abs/2307.09383`

- $\mathrm{F}^{\circ}_{\mathrm{eff}}$: a *system F*-style calculus with subkinding-based linear types and row-based effect types. Core calculus of LINKS (to some extent). Metatheory: type soundness + runtime linearity safety.
- $\mathrm{Q}^{\circ}_{\mathrm{eff}}$: an *ML*-style calculus with linear types and effect types both based on *qualified types*. Full type inference with principal types. Deterministic constraint solving. Better accuracy enabled by effect subtyping.

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Potential future work:

- CFL with modalities.
- Algebraic subtyping for linearity (and effects).
- Shallow handlers.

Thank you!

Takeaway: consider tracking control-flow linearity when having both linear types and effect handlers!