# Soundly Handling Linearity

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The University of Edinburgh

TUPLE, the University of Edinburgh, 21 Feb 2024

(Joint work with Daniel Hillerström, Sam Lindley, and J. Garrett Morris)

## linear types

*Picture by Xueying Qin*

# Linear Types vs Effect Handlers

**linear types**

RUST, HASKELL

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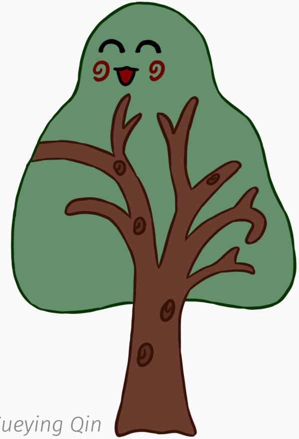

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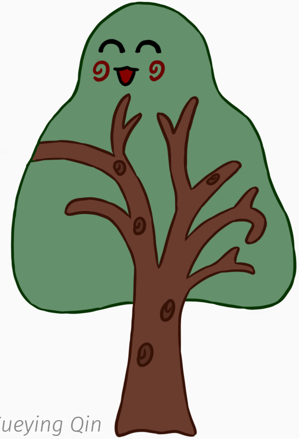

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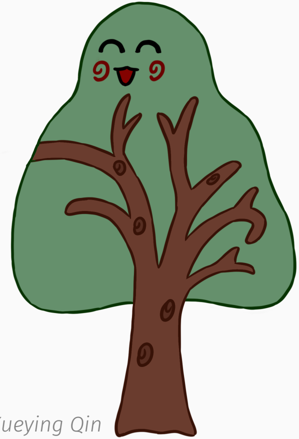

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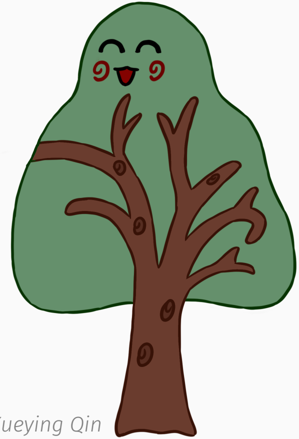

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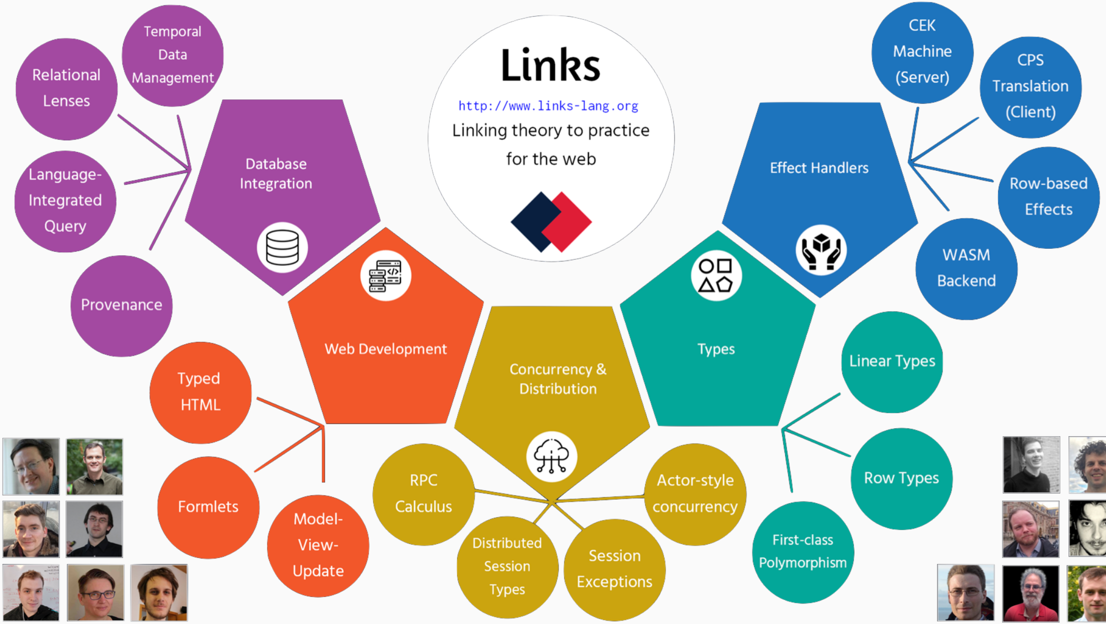
# Overview

In this talk, I will

- give a quick introduction to LINKS
- explain what are *linear types* (and *session types*)
- explain what are *algebraic effects and handlers*
- break LINKS by using them together
- fix LINKS by tracking *control-flow linearity*
- show how to further improve LINKS

Feel free to interrupt me at any time!

# Introduction to LINKS




Picture by Simon Fowler

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links> id("Hello world!"); # instantiate a to String
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links> typename Channel = !Int.End;  # an alias for a primitive linear type
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Type error: Variable ch has linear type `Channel' but is used 2 times.

links> fun discardLin(ch:Channel) { 42 };
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  println(intToString(i))
}
```

# Connect Sender and Receiver

Fork the receiver and pass the dual channel endpoint to the sender.

```
links> { var c = fork(receiver); sender(c) };
42
() : ()
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Type error: Variable c of linear type `!Int.End' is used in a non-linear function.
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links> { var c = fork(receiver);
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Type error: Variable f has linear type `() -@ ()' but is used 2 times.
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# Algebraic Effects and Handlers

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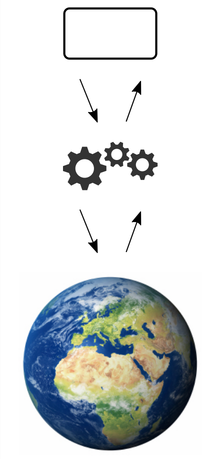

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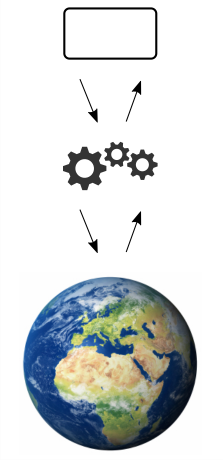

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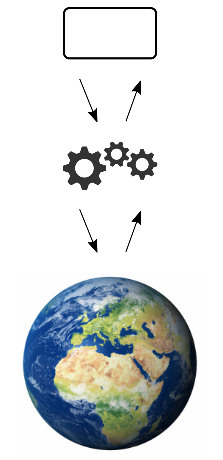

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Typically ad hoc and hard-wired

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Growing industrial interest

| | | |
|---|---|---|
| GitHub | `semantic` | Code analysis library ($>$ 25 million repositories) |
| | React | JavaScript UI library ($>$ 2 million websites) |
| Uber | Pyro | Statistical inference (10% ad spend saving) |

*Table from Sam Lindley*

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I don't want to print :(
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fun ndprinter() { var i = if (do Choose) then 42 else 84; printInt(i) }


links> handle (ndprinter())
       { case <Choose => r> -> r(true) };           # one-shot handler
       # fun r(b) { var i = if (b) then 42 else 84; printInt(i) }
42
```

# Implementing Nondeterminism

```
sig ndprinter   : () { Choose: () => Bool | _ }~> ()
# the function type is decorated with an effect type { Choose: () => Bool | _ }
# which means this function may use the Choose operation
# which takes no parameter and returns a boolean value
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# Breaking LINKS

# Well-typed programs in Links CAN go wrong !

We can break Links by duplicating a linear channel with multi-shot effect handlers!

## Well-typed programs in LINKS CAN go wrong !

We can break LINKS by duplicating a linear channel with multi-shot effect handlers!

```
sig ndsender    : (!Int.End) { Choose: () => Bool | _ }~> ()
fun ndsender(c) {
  var x  = if (do Choose) then 42 else 84; # choose an integer to send
  var c' = send(x, c);                     # send x to c
  close(c')                                # close the remaining c'
}
```

## Well-typed programs in LINKS CAN go wrong !

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  close(c')                                # close the remaining c'
}

links> handle ({ var c = fork(receiver); ndsender(c) })
       { case <Choose => r> -> r(true); r(false) };
42***: Internal Error in evalir.ml : NotFound chan_3 while interpreting.
```

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links> handle ({ var c = fork(receiver); ndsender(c) })
       { case <Choose => r> -> r(true); r(false) };
42***: Internal Error in evalir.ml : NotFound chan_3 while interpreting.

continuation of Choose:
       fun r(b) { var x  = if (b) then 42 else 84;
                  var c' = send(x, c); # c is captured in the continuation
                  close(c') }          # it is closed when excuting r(true)
```

# Why doesn't LINKS reject us using r twice?

```
sig ndsender    : (!Int.End) { Choose: () => Bool | _ }~> ()
fun ndsender(c) {
  var x  = if (do Choose) then 42 else 84;
  var c' = send(x, c);
  close(c')
}
```

```
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fun r(b) { var x  = if (b) then 42 else 84; var c' = send(x, c); close(c') }
```

One point of view:

Conventional linear type systems only track ***value linearity***, i.e., linearity of primitive values, pairs, functions, etc. They already exist in the source code in the form of values. However, the continuation function `r` of `Choose` is dynamically created during evaluation.

# Why doesn't LINKS reject us using r twice?

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fun ndsender(c) {
  var x  = if (do Choose) then 42 else 84;
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  close(c')
}

continuation of Choose:
fun r(b) { var x  = if (b) then 42 else 84; var c' = send(x, c); close(c') }
```

Another point of view:

Conventional linear type systems assume that the ***control flow*** goes normally from the beginning to the end. It only enters the continuation of `do Choose` once.

However, effect handlers allow the control flow to jump back to `do Choose`.

# Why doesn't LINKS reject us using r twice?

```
sig ndsender    : (!Int.End) { Choose: () => Bool | _ }~> ()
fun ndsender(c) {
  var x  = if (do Choose) then 42 else 84;
  var c' = send(x, c);
  close(c')
}

continuation of Choose:
fun r(b) { var x  = if (b) then 42 else 84; var c' = send(x, c); close(c') }
```

Solution: track ***control-flow linearity*** in addition to value linearity.

- A *control-flow-linear* operation: the control flow must enter its cont exactly once.
- A *control-flow-unlimited* operation: the control flow may enter its cont any times.

# Fixing Links

# Tracking Control-Flow Linearity in Links

```
sig ndsender :
  (!Int.End)
    { Choose: () => Bool
    | _ }~>
  ()
fun ndsender(c) {
  # by default, the control-flow linearity is unlimited
  var x  = if (do Choose)
           then 42 else 84;
  var c' = send(x, c);
  close(c')
}
```

Ill-typed because we cannot use the linear variable `c` in a control-flow-unlimited environment after the control-flow-unlimited operation `Choose`.

# Tracking Control-Flow Linearity in LINKS

```
sig ndsender :
  (!Int.End)
    { Choose: () =@ Bool      # annotate Choose as control-flow linear 🍭
    | _ }~>
  ()
fun ndsender(c) {
  # by default, the control-flow linearity is unlimited
  var x  = if (do Choose)
           then 42 else 84;
  var c' = send(x, c);
  close(c')
}
```

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  (!Int.End)
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    | _ }~>
  ()
fun ndsender(c) {
  # by default, the control-flow linearity is unlimited
  var x  = if (lindo Choose) # invoke a control-flow-linear operation
           then 42 else 84;
  var c' = send(x, c);
  close(c')
}
```

# Tracking Control-Flow Linearity in LINKS

```
sig ndsender :
  (!Int.End)
    { Choose: () =@ Bool      # annotate Choose as control-flow linear 🍭
    | _ }~>
  ()
fun ndsender(c) {
  xlin;                       # switch the control-flow linearity to linear
  var x  = if (lindo Choose) # invoke a control-flow-linear operation
           then 42 else 84;
  var c' = send(x, c);
  close(c')
}
```

# Tracking Control-Flow Linearity in Links

```
sig ndsender :
  (!Int.End)
    { Choose: () =@ Bool      # annotate Choose as control-flow linear 🍭
    | _::Lin }~>              # require other potential operations to be linear
  ()
fun ndsender(c) {
  xlin;                       # switch the control-flow linearity to linear
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           then 42 else 84;
  var c' = send(x, c);
  close(c')
}
```

Well-typed since we are using a linear variable `c` and a control-flow-linear operation `Choose` in a control-flow-linear environment.

# Back to the Full Example

```
sig receiver    : (?Int.End) { | _ }~> ()
fun receiver(c) { var (i, c') = receive(c); close(c'); printInt(i) }

sig ndsender    : (!Int.End) {Choose: () => Bool | _ }~> ()
fun ndsender(c) { close(send(if (do Choose) 42 else 84, c)) }

links> handle ({ var c = fork(receiver); ndsender(c) })
       { case <Choose => r> -> r(true); r(false) };
42***: Internal Error in evalir.ml : NotFound chan_3 while interpreting.
```

# Back to the Full Example

```
sig receiver    : (?Int.End) { | _::Lin }~> ()
fun receiver(c) { xlin; var (i, c') = receive(c); close(c'); printInt(i) }

sig ndsender    : (!Int.End) {Choose: () =@ Bool | _::Lin }~> ()
fun ndsender(c) { xlin; close(send(if (lindo Choose) 42 else 84, c)) }

links> handle ({ xlin; var c = fork(receiver); ndsender(c) })
       { case <Choose => r> -> xlin; r(true); r(false) };
Type Error: ... =@ does not match => ...
```

# Back to the Full Example

```
sig receiver    : (?Int.End) { | _::Lin }~> ()
fun receiver(c) { xlin; var (i, c') = receive(c); close(c'); printInt(i) }

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links> handle ({ xlin; var c = fork(receiver); ndsender(c) })
       { case <Choose =@ r> -> xlin; r(true); r(false) };
       # use =@ for handler clauses of control-flow-linear operations
Type Error: Variable r has linear type but is used 2 times.
```

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sig receiver    : (?Int.End) { | _::Lin }~> ()
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Type Error: Variable r has linear type but is used 2 times.
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Well-typed programs cannot go wrong!

# Beyond LINKS

# Restriction of Linear Types and Control-Flow Linearity in Links

We lose *principal types*. As a result, we need to have different versions of (almost) the same function with different types, which breaks modularity and reusability.

# Restriction of Linear Types and Control-Flow Linearity in Links

We lose *principal types*. As a result, we need to have different versions of (almost) the same function with different types, which breaks modularity and reusability.

Consider the verbose identity function

```
fun verboseId(x) {do Print("id"); x}
```

# Restriction of Linear Types and Control-Flow Linearity in Links

We lose *principal types*. As a result, we need to have different versions of (almost) the same function with different types, which breaks modularity and reusability.

Consider the verbose identity function

```
fun verboseId(x) {do Print("id"); x}
```

Without linear types, we only need one version of it with the type

```
sig verboseId : (a) { Print : (String) => () | _ }-> a
fun verboseId(x) {do Print("id"); x}
```

# Restriction of Linear Types and Control-Flow Linearity in Links

We lose *principal types*. As a result, we need to have different versions of (almost) the same function with different types, which breaks modularity and reusability.

Consider the verbose identity function

```
fun verboseId(x) {do Print("id"); x}
```

With linear types, we have two versions

```
sig verboseId : (a::Any) { Print : (String) => () | _ }-> a::Any
fun verboseId(x) {do Print("id"); x}
sig verboseId : (a::Any) { Print : (String) => () | _ }-@ a::Any
linfun verboseId(x) {do Print("id"); x}
```

# Restriction of Linear Types and Control-Flow Linearity in Links

We lose *principal types*. As a result, we need to have different versions of (almost) the same function with different types, which breaks modularity and reusability.

Consider the verbose identity function

```
fun verboseId(x) {do Print("id"); x}
```

Further with control-flow linearity, we have six versions

```
sig verboseId : (a) { Print : (String) => () | _ }-> a
fun verboseId(x) {do Print("id"); x}
sig verboseId : (a) { Print : (String) =@ () | _ }-> a
fun verboseId(x) {lindo Print("id"); x}
sig verboseId : (a::Any) { Print : (String) =@ () | _::Lin }-> a::Any
fun verboseId(x) {xlin; lindo Print("id"); x}
linfun ... linfun ... linfun ...
```

# Principal Types with Constraints

We can restore principal types in LINKS using constraints / qualified types

```
sig verboseId : a { Print : (String) =>^ϕ () | ρ }->^ϕ' a with (a ≤ ϕ, a ≤ ρ)
fun verboseId(x) {do Print("id"); x}
```

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We can restore principal types in LINKS using constraints / qualified types

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sig verboseId : a { Print : (String) =>ϕ () | ρ }->ϕ' a with (a ≤ ϕ, a ≤ ρ)
fun verboseId(x) {do Print("id"); x}
```

`->`$^{\phi'}$ can be instantiated to either `->` or `-@`

`=>`$^{\phi}$ can be instantiated to either `=>` or `=@` satisfying the condition that when `a` is a linear type, it must be `=@`

$\rho$ can either have kind `Lin` or `Any` satisfying the condition that when `a` is a linear type, it must have kind `Lin`

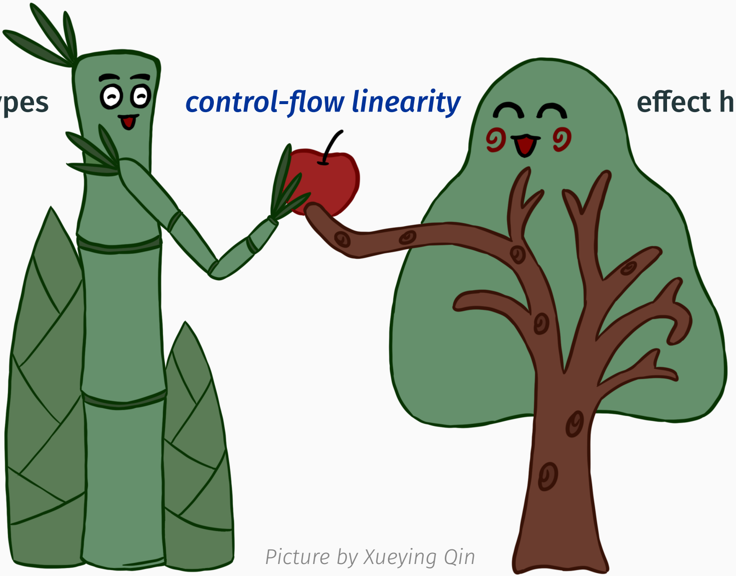
# More in the Paper

$F^{\circ}_{\text{eff}}$
- *system-F* style
- *subkinding*-based linear types [Mazurak et al. 2010]
- *row*-based effect types [Hillerström and Lindley 2016]
- implementation in LINKS
- metatheory (type soundness and runtime linearity safety)

$Q^{\circ}_{\text{eff}}$
- *ML* style
- ***qualified* linear types** based on QUILL [Morris 2016]
- ***qualified* effect types** based on ROSE [Morris and McKinna 2019]
- **type inference** with principal types
- deterministic constraint solving
- metatheory (soundness and completeness of type inference)

Takeaway: consider tracking *control-flow linearity* when having both linear types and effect handlers in your languages!

**linear types** **control-flow linearity** **effect handlers**



*Picture by Xueying Qin*