Tracking Linear Continuations for Effect Handlers

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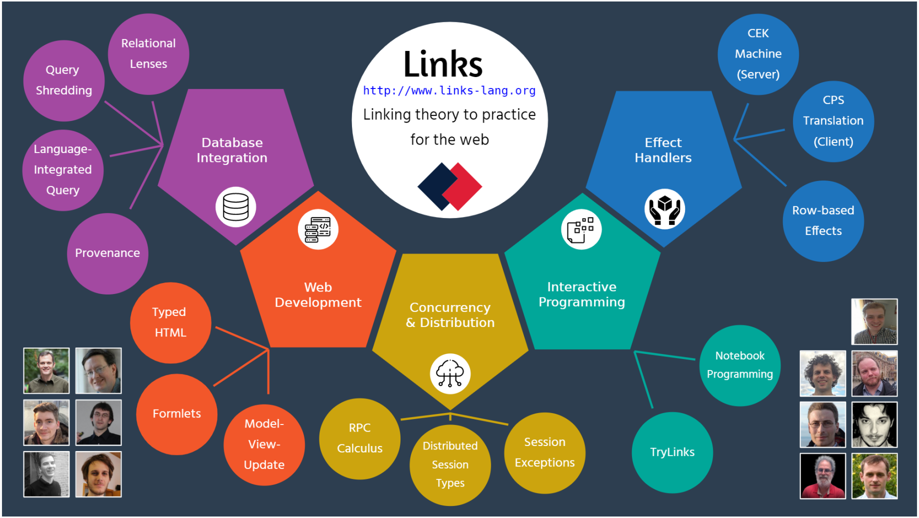
Links



Picture by Simon Fowler

Session Types in Links

Links session types:

- `!A.S` : send a value of type `A`, then continue as `S`
- `?A.S` : receive a value of type `A`, then continue as `S`
- `End` : no communication

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Primitive operations on session-typed channels:

```
send    : ∀ a (b::Session) . (a, !a.b) → b
receive : ∀ a (b::Session) . (?a.b) → (a, b)
fork    : ∀   (a::Session) . (a → ()) → ~a
close   : End → ()
```

Session Types in Links are Sound

```
sig sender       : (!Int.End) → ()
fun sender(ch)   { var ch = send(42, ch); close(ch) }
```

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sig sender        : (!Int.End) ⟶ ()
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sig receiver      : (?Int.End) ⟶ ()
fun receiver(ch) { var (i, ch) = receive(ch); close(ch); printInt(i) }
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```

```
links> { var ch = fork(receiver); sender(ch) };
42
```

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sig receiver      : (?Int.End) → ()
fun receiver(ch) { var (i, ch) = receive(ch); close(ch); printInt(i) }
```

```
links> { var ch = fork(receiver); sender(ch) };
42
```

```
links> { var ch = fork(receiver); var ch = send(42, ch); close(ch);
                                                          close(ch) };
<stdin>:1: Type error: Variable ch has linear type
    `End'
but is used 2 times.
In expression: var ch = send(42, ch);.
```

Session Types in Links are Sound

```
sig sender        : (!Int.End) ⟶ ()
fun sender(ch)    { var ch = send(42, ch); close(ch) }
sig receiver      : (?Int.End) ⟶ ()
fun receiver(ch) { var (i, ch) = receive(ch); close(ch); printInt(i) }
```

```
links> { var ch = fork(receiver); sender(ch) };
42
```

```
links> { var ch = fork(receiver); var ch = send(42, ch); close(ch);
                                  var ch = send(42, ch); close(ch) };
<stdin>:1: Type error: The function
    `send'
has type
    `(Int, !(Int).a::Session) ~b⟶ a::Session'
while the arguments passed to it have types
    `Int' and `End'
In expression: send(42, ch).
```

Session Types in Links are Sound

```
sig sender        : (!Int.End) → ()
fun sender(ch)    { var ch = send(42, ch); close(ch) }
sig receiver      : (?Int.End) → ()
fun receiver(ch) { var (i, ch) = receive(ch); close(ch); printInt(i) }
```

```
links> { var ch = fork(receiver); sender(ch) };
42
```

```
links> { var ch = fork(receiver);
         var f = fun(){ var ch = send(42, ch); close(ch) }; f(); f() };
<stdin>:1: Type error: Variable ch of linear type ~?(Int).End is used in a
    non-linear function literal.
In expression: fun(){var ch = send(42, ch); close(ch)}.
```

Session Types in Links are Sound

```
sig sender       : (!Int.End) → ()
fun sender(ch)   { var ch = send(42, ch); close(ch) }
sig receiver     : (?Int.End) → ()
fun receiver(ch) { var (i, ch) = receive(ch); close(ch); printInt(i) }
```

```
links> { var ch = fork(receiver); sender(ch) };
42
```

```
links> { var ch = fork(receiver);
         var f = linfun(){ var ch = send(42, ch); close(ch) }; f(); f() };
<stdin>:1: Type error: Variable f has linear type
    `() ~a~@ ()'
but is used 2 times.
In expression: var f = linfun(){var ch = send(42, ch); close(ch)};.
```

Effect Handlers in Links

```
sig choose : () { Choose: () ↠ Bool }⟶ ()
fun choose() { var i = if (do Choose) 42 else 1; printInt(i) }
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```
links> handle (choose())
         { case <Choose ⇒ r> ⟶ r(true) }
42
```

Effect Handlers in Links

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sig choose : () { Choose: () ↠ Bool }⟶ ()
fun choose() { var i = if (do Choose) 42 else 1; printInt(i) }
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```
links> handle (choose())
        { case <Choose ⇒ r> ⟶ r(true) }
42
```

```
links> handle (choose())
        { case <Choose ⇒ r> ⟶ r(true); r(false) }
42 1
```

Well Typed Programs Can Go Wrong in Links

```
sig sender2     : (!Int.End) { Choose: () ↠ Bool }⟶ ()
fun sender2(ch) { var i = if (do Choose) 42 else 1;
                  var ch = send(i, ch); close(ch) }
```

Well Typed Programs Can Go Wrong in Links

```
sig sender2     : (!Int.End) { Choose: () ↠ Bool }⟶ ()
fun sender2(ch) { var i = if (do Choose) 42 else 1;
                  var ch = send(i, ch); close(ch) }


links> handle ({ var ch = fork(receiver); sender2(ch) })
        { case <Choose ⇒ r> ⟶ r(true) }
42
```

Well Typed Programs Can Go Wrong in Links [1,2]

```
sig sender2     : (!Int.End) { Choose: () ↠ Bool }→ ()
fun sender2(ch) { var i = if (do Choose) 42 else 1;
                  var ch = send(i, ch); close(ch) }
```

```
links> handle ({ var ch = fork(receiver); sender2(ch) })
        { case <Choose ⇒ r> → r(true) }
42
```

```
links> handle ({ var ch = fork(receiver); sender2(ch) })
        { case <Choose ⇒ r> → r(true); r(false) }
***: Internal Error in evalir.ml (Please report as a bug): NotFound
 chan_7 (in Hashtbl.find) while interpreting.
```

[1] https://github.com/links-lang/links/issues/544
[2] Emrich and Hillerström, "Broken Links (Presentation)", 2020.

Our Main Contributions

- F°_{eff}: an extension of system F with correct interaction between linear types and effect handlers.
- Prove the safety of F°_{eff}.
- Q°_{eff}: a ML-variant of F°_{eff} with full type inference based on qualified types.

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- Prove the safety of F°_{eff}.
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F° : System F with Linear Types[3]

Value types	$A, B ::= \alpha \mid A \to^{Y} B \mid \forall^{Y} \alpha^{K}.A$	Kinds $K ::=$
Linearity	$Y ::= \bullet \mid \circ$	Type^{Y}
Type contexts	$\Gamma ::= \cdot \mid \Gamma, x : A$	
Kind contexts	$\Delta ::= \cdot \mid \Delta, \alpha : K$	
Terms	$M, N ::= x \mid \lambda^{Y} x^{A}.M \mid \Lambda^{Y} \alpha^{K}.M \mid M\, N \mid M\, A$	

[3]Mazurak, Zhao, and Zdancewic, “Lightweight Linear Types in System F^{o}”, 2010.

F° Examples

$$id = \Lambda^\bullet \alpha^{\mathsf{Type}^\circ}.\lambda^\bullet x^\alpha.x : \forall^\bullet \alpha^{\mathsf{Type}^\circ}.\alpha \to^\bullet \alpha$$

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$$id\ Int\ 42 : Int$$

F° Examples

$$id = \Lambda^{\bullet}\alpha^{\mathsf{Type}^{\circ}}.\lambda^{\bullet}x^{\alpha}.x : \forall^{\bullet}\alpha^{\mathsf{Type}^{\circ}}.\alpha \to^{\bullet} \alpha$$

F° has subkinding $\mathsf{Type}^{\bullet} \leq \mathsf{Type}^{\circ}$:

$$id\ Int\ 42 : Int$$

Suppose we still have built-in session types, and omit the linearity annotations on terms and types when it is $\bullet$.

$$sendAndClose = \lambda f^{!Int.End}.\lambda^{\circ}x^{Int}.close\,(send\,(x, f)) : (!Int.End) \to Int \to^{\circ} ()$$

F_{eff} : System F with Effect Handlers[4]

		Kinds $K ::=$
Value types	$A, B ::= \alpha \mid A \to C \mid \forall \alpha^K.C$	Type
Computation types	$C, D ::= A \,!\, E$	Comp
Effect types	$E ::= \{R\}$	Effect
Row types	$R ::= \ell : P; R \mid \mu \mid \cdot$	$\mathsf{Row}_{\mathcal{L}}$
Presence types	$P ::= \mathsf{Abs} \mid A \twoheadrightarrow B \mid \theta$	Presence
Handler types	$F ::= C \rightrightarrows D$	Handler
Types	$T ::= A \mid R \mid P$	
Type contexts	$\Gamma ::= \cdot \mid \Gamma, x : A$	
Kind contexts	$\Delta ::= \cdot \mid \Delta, \alpha : K$	
Values	$V, W ::= x \mid \lambda x^A.M \mid \Lambda \alpha^K.M$	
Computations	$M, N ::= V\,W \mid V\,T \mid (\mathbf{return}\ V)^E \mid (\mathbf{do}\ \ell\ V)^E$	
	$\mid\ \mathbf{let}\ x \leftarrow M\ \mathbf{in}\ N \mid \mathbf{handle}\ M\ \mathbf{with}\ H$	
Handlers	$H ::= \{\mathbf{return}\ x \mapsto M\} \mid \{\ell\ p\ r \mapsto M\} \uplus H$	

[4] Hillerström, Lindley, and Atkey, "Effect handlers via generalised continuations", 2020.

$\mathrm{F_{eff}} + \mathrm{F}^{\circ}$ is BROKEN

Define $M; N \equiv \mathbf{let}\ \_ \leftarrow M\ \mathbf{in}\ N$.

Assuming a global channel $f : \mathit{End}$, we have:

$$\mathbf{handle}\ \overbrace{(\mathbf{do}\ \mathit{Choose}\ (); \mathit{close}\ f)}^{()\,!\,\{\mathit{Choose}:()\twoheadrightarrow \mathit{Bool}\}}\ \mathbf{with}\ \overbrace{\{\mathit{Choose}\ \_\ \underset{\mathit{Bool}\to()\,!\,\{\}}{r} \mapsto r\ \mathit{true}; r\ \mathit{false}\}}^{()\,!\,\{\mathit{Choose}:()\twoheadrightarrow \mathit{Bool}\}\rightrightarrows()\,!\,\{\}}$$

F_{eff} + F° is BROKEN

Define $M; N \equiv \mathbf{let}\ \_ \leftarrow M\ \mathbf{in}\ N$.

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$$\leadsto \quad (r\ \mathit{true}; r\ \mathit{false})[(\lambda\_.\mathit{close}\ f)/r]$$

$\mathrm{F}_{\mathrm{eff}} + \mathrm{F}^{\circ}$ is BROKEN

Define $M; N \equiv \mathbf{let}\ \_ \leftarrow M\ \mathbf{in}\ N$.

Assuming a global channel $f : \mathit{End}$, we have:

$$
\begin{aligned}
& \mathbf{handle}\ \overbrace{(\mathbf{do}\ \mathit{Choose}\ (); \mathit{close}\ f)}^{()\,!\,\{\mathit{Choose}:() \twoheadrightarrow \mathit{Bool}\}}\ \mathbf{with}\ \overbrace{\{\mathit{Choose}\ \_\ \underset{\mathit{Bool} \to ()\,!\,\{\}}{r} \mapsto r\ \mathit{true}; r\ \mathit{false}\}}^{()\,!\,\{\mathit{Choose}:() \twoheadrightarrow \mathit{Bool}\} \rightrightarrows ()\,!\,\{\}} \\
\rightsquigarrow\quad & (r\ \mathit{true}; r\ \mathit{false})[(\lambda\_.\mathit{close}\ f)/r] \\
=\quad & \mathit{close}\ f; \mathit{close}\ f
\end{aligned}
$$

f is closed twice!

$\mathrm{F}_{\mathrm{eff}} + \mathrm{F}^{\circ} = \mathrm{F}^{\circ}_{\mathrm{eff}}$

Value types	$A, B ::= \alpha \mid A \to^{Y} C \mid \forall^{Y} \alpha^{K}.C$
Computation types	$C, D ::= A \,!\, E$
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Row types	$R ::= \ell : P; R \mid \mu \mid \cdot$
Presence types	$P ::= \mathsf{Abs} \mid A \twoheadrightarrow^{Y} B \mid \theta$
Handler types	$F ::= C \rightrightarrows D$
Types	$T ::= A \mid R \mid P$
Label sets	$\mathcal{L} ::= \emptyset \mid \{\ell\} \uplus \mathcal{L}$
Linearity	$Y ::= \bullet \mid \circ$
Type contexts	$\Gamma ::= \cdot \mid \Gamma, x : A$
Kind contexts	$\Delta ::= \cdot \mid \Delta, \alpha : K$
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Handlers	$H ::= \{\mathbf{return}\ x \mapsto M\} \mid \{\ell\ p\ r \mapsto M\} \uplus H$

Kinds $K ::=$

Type^{Y}

Comp

Effect

$\mathsf{Row}^{Y}_{\mathcal{L}}$

$\mathsf{Presence}^{Y}$

$\mathsf{Handler}$

Fixing $\mathrm{F_{eff}} + \mathrm{F}^\circ$

It would be great to know that r should be a linear function:

$$\textbf{handle}\ \overbrace{(\textbf{do}\ \mathit{Choose}\ (); \mathit{close}\ f)}^{()\,!\,\{\mathit{Choose}:()\twoheadrightarrow\mathit{Bool}\}}\ \textbf{with}\ \overbrace{\{\mathit{Choose}\ \_\ \underset{\mathit{Bool}\to^\circ()\,!\,\{\}}{r} \mapsto r\ \mathit{true}; r\ \mathit{false}\}}^{()\,!\,\{\mathit{Choose}:()\twoheadrightarrow\mathit{Bool}\}\rightrightarrows()\,!\,\{\}}$$

Fixing $\mathrm{F}_{\text{eff}} + \mathrm{F}^{\circ}$

We could look at the effect signature of *Choose*:

$$\textbf{handle}\ \overbrace{(\textbf{do}\ \mathit{Choose}\,();\mathit{close}\,f)}^{()\,!\,\{\mathit{Choose}:()\twoheadrightarrow^{\circ}\mathit{Bool}\}}\ \textbf{with}\ \overbrace{\{\mathit{Choose}\ \_\ \underset{\mathit{Bool}\to^{\circ}()\,!\,\{\}}{r}\mapsto r\,\mathit{true};r\,\mathit{false}\}}^{()\,!\,\{\mathit{Choose}:()\twoheadrightarrow^{\circ}\mathit{Bool}\}\rightrightarrows()\,!\,\{\}}$$

Fixing $F_{eff} + F^{\circ}$

Notice that *close* f uses a linear variable f:

$$\mathbf{handle}\ \overbrace{(\mathbf{do}\ \mathit{Choose}\ (); \mathit{close}\ \underbrace{f}_{f:\mathit{End}\in\Gamma})}^{()\,!\,\{\mathit{Choose}:()\twoheadrightarrow^{\circ}\mathit{Bool}\}}\ \mathbf{with}\ \overbrace{\{\mathit{Choose}\ \_\ \underbrace{r}_{\mathit{Bool}\to^{\circ}()\,!\,\{\}} \mapsto r\ \mathit{true}; r\ \mathit{false}\}}^{()\,!\,\{\mathit{Choose}:()\twoheadrightarrow^{\circ}\mathit{Bool}\}\rightrightarrows()\,!\,\{\}}$$

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Core idea: add linearity annotations on effect signatures, and track the linearity information while typing.

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Core idea: add linearity annotations on effect signatures, and track the linearity information while typing.

The linearity Y in $\mathit{Choose} : () \twoheadrightarrow^{Y} \mathit{Bool}$ reflects *control-flow linearity*, i.e. the usage restriction on its context / continuation.

Duality between value linearity and control-flow linearity

For $V : (A : \mathrm{Type}^Y)$, Y restricts the linearity of *the value itself*

- $Y = \circ$
  - V is guaranteed to be used linearly
  - V may contain linear resources
- $Y = \bullet$
  - no guarantee on the usage of V
  - V must not contain linear resources

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less restriction on itself ($\mathsf{Type}^\bullet \leq \mathsf{Type}^\circ$)

Duality between value linearity and control-flow linearity

For $\mathbf{do}\ \ell\, V : A\,!\,\{\ell : A' \twoheadrightarrow^{Y} B'\}$, Y restricts the linearity of *its context*

- $Y = \circ$
  - ℓ is guaranteed to be handled linearly
  - ℓ's continuation may contain linear resources
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more restriction on its context

However, we cannot upcast $\ell : A' \twoheadrightarrow^{\circ} B'$ to $\ell : A' \twoheadrightarrow^{\bullet} B'$ because it would break the safety of handling.

Duality between value linearity and control-flow linearity

Instead, we can upcast the kind of row types.

For $M : A\,!\,\{(R : \mathrm{Row}_{\emptyset}{}^{Y})\}$, Y restricts the linearity of *its context*

- $Y = \circ$
  - operations in M are guaranteed to be handled linearly
  - M's continuation may contain linear resources
- $Y = \bullet$
  - no guarantee on the handling of operations in M
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more restriction on its context

$$\frac{}{\vdash \bullet \leq \circ} \qquad \frac{\vdash Y \leq Y'}{\vdash \mathrm{Type}^{Y} \leq \mathrm{Type}^{Y'}} \qquad \frac{\vdash Y' \leq Y}{\vdash \mathrm{Presence}^{Y} \leq \mathrm{Presence}^{Y'}} \qquad \frac{\vdash Y' \leq Y}{\vdash \mathrm{Row}_{\mathcal{L}}{}^{Y} \leq \mathrm{Row}_{\mathcal{L}}{}^{Y'}}$$

Tracking Control-Flow Linearity

The evaluation context tells us that continuations consist of only *sequencing* and *handling*.

$$\text{E-OP} \quad \textbf{handle}\ \mathcal{E}[\textbf{do}\ \ell\ V]\ \textbf{with}\ H \leadsto N[V/p, (\lambda y.\textbf{handle}\ \mathcal{E}[\textbf{return}\ y]\ \textbf{with}\ H)/r]$$
$$\text{where } \ell \notin \mathsf{bl}(\mathcal{E}) \text{ and } (\ell\ p\ r \mapsto N) \in H$$

$$\text{Evaluation context } \mathcal{E} ::= [\ ] \mid \textbf{let}\ x \leftarrow \mathcal{E}\ \textbf{in}\ N \mid \textbf{handle}\ \mathcal{E}\ \textbf{with}\ H$$

Tracking Control-Flow Linearity

The evaluation context tells us that continuations consist of only *sequencing* and *handling*.

$$\textsc{E-Op}\quad \textbf{handle}\ \mathcal{E}[\textbf{do}\ \ell\ V]\ \textbf{with}\ H \rightsquigarrow N[V/p, (\lambda y.\textbf{handle}\ \mathcal{E}[\textbf{return}\ y]\ \textbf{with}\ H)/r]$$
$$\text{where } \ell \notin \mathsf{bl}(\mathcal{E}) \text{ and } (\ell\ p\ r \mapsto N) \in H$$

$$\text{Evaluation context } \mathcal{E} ::= [\ ] \mid \textbf{let}\ x \leftarrow \mathcal{E}\ \textbf{in}\ N \mid \textbf{handle}\ \mathcal{E}\ \textbf{with}\ H$$

As deep handlers are always recursive, they cannot use any linear resource.

T-Handler

$$C = A\,!\,\{(\ell_i : A_i \twoheadrightarrow^{Y_i} B_i)_i; R\} \qquad D = B\,!\,\{(\ell_i : P)_i; R\}$$
$$H = \{\textbf{return}\ x \mapsto M\} \uplus \{\ell_i\ p_i\ r_i \mapsto N_i\}_i$$
$$\frac{\underset{\text{all types in }\Gamma\text{ are unlimited}}{\Delta \vdash \Gamma : \bullet} \qquad \Delta; \Gamma, x : A \vdash M : D \qquad [\Delta; \Gamma, p_i : A_i, r_i : B_i \to^{Y_i} D \vdash N_i : D]_i}{\Delta; \Gamma \vdash H : C \rightrightarrows D}$$

Tracking Control-Flow Linearity (Cont.)

Sequencing has a real influence on control-flow linearity.

We can make use of the kinding relation of row types:

T-SeqEq

$$\frac{\Delta;\Gamma_1 \vdash M : A \,!\, \{R\} \qquad \Delta;\Gamma_2, x : A \vdash N : B \,!\, \{R\} \qquad \Delta \vdash (\Gamma_2, x : A) : Y \qquad \Delta \vdash R : \mathrm{Row}^{Y}}{\Delta;\Gamma_1 + \Gamma_2 \vdash \mathbf{let}\ x \leftarrow M\ \mathbf{in}\ N : B \,!\, \{R\}}$$

$Y{=}\circ$: $(\Gamma_2, x : A)$ may contain linear vars

$Y{=}\bullet$: $(\Gamma_2, x : A)$ only contains unlimited vars

$Y{=}\circ$: R only contains linear ops ($\twoheadrightarrow^{\circ}$)

$Y{=}\bullet$: R may contain unlimited ops ($\twoheadrightarrow^{\bullet}$)

Sound, but not Precise Enough?

$$\frac{\cdot;\cdot \vdash \mathbf{do}\ \mathit{Choose}\,() : ()\,!\,\{R\} \qquad \cdot; f : \mathit{End} \vdash \mathbf{do}\ \mathit{Close}\,f : ()\,!\,\{R\} \qquad \cdot \vdash (f : \mathit{End}) : \circ \qquad \cdot \vdash R : \mathrm{Row}_{\emptyset}{}^{\circ}}{\cdot; f : \mathit{End} \vdash \mathbf{do}\ \mathit{Choose}\,(); \mathbf{do}\ \mathit{Close}\,f : ()\,!\,\{R\}}$$

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$R = \{\mathit{Choose} : () \twoheadrightarrow^{\circ} \mathit{Bool}; \mathit{Close} : \mathit{End} \twoheadrightarrow^{\circ} ()\}$ is well-typed but too restrictive

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$$\frac{\cdot;\cdot \vdash \mathbf{do}\ \mathit{Choose}\,() : ()\,!\,\{R\} \qquad \cdot; f : \mathit{End} \vdash \mathbf{do}\ \mathit{Close}\, f : ()\,!\,\{R\} \qquad \cdot \vdash (f : \mathit{End}) : \circ \qquad \cdot \vdash R : \mathrm{Row}_{\emptyset}{}^{\circ}}{\cdot; f : \mathit{End} \vdash \mathbf{do}\ \mathit{Choose}\,();\mathbf{do}\ \mathit{Close}\, f : ()\,!\,\{R\}}$$

$R = \{\mathit{Choose} : () \twoheadrightarrow^{\circ} \mathit{Bool}; \mathit{Close} : \mathit{End} \twoheadrightarrow^{\circ} ()\}$ is well-typed but too restrictive

$R = \{\mathit{Choose} : () \twoheadrightarrow^{\circ} \mathit{Bool}; \mathit{Close} : \mathit{End} \twoheadrightarrow^{\bullet} ()\}$ is more precise but ill-typed

Sound, but not Precise Enough?

$$\frac{\cdot;\cdot \vdash \mathbf{do}\ \mathit{Choose}\ () : ()\ !\ \{R\} \qquad \cdot; f : \mathit{End} \vdash \mathbf{do}\ \mathit{Close}\ f : ()\ !\ \{R\} \qquad \cdot \vdash (f : \mathit{End}) : \circ \qquad \cdot \vdash R : \mathrm{Row}_{\emptyset}{}^{\circ}}{\cdot; f : \mathit{End} \vdash \mathbf{do}\ \mathit{Choose}\ (); \mathbf{do}\ \mathit{Close}\ f : ()\ !\ \{R\}}$$

$R = \{\mathit{Choose} : () \twoheadrightarrow^{\circ} \mathit{Bool}; \mathit{Close} : \mathit{End} \twoheadrightarrow^{\circ} ()\}$ is well-typed but too restrictive

$R = \{\mathit{Choose} : () \twoheadrightarrow^{\circ} \mathit{Bool}; \mathit{Close} : \mathit{End} \twoheadrightarrow^{\bullet} ()\}$ is more precise but ill-typed

$$\frac{\cdot;\cdot \vdash \mathbf{do}\ \mathit{Choose}\ () : ()\ !\ \{R_1\} \qquad \cdot; f : \mathit{End} \vdash \mathbf{do}\ \mathit{Close}\ f : ()\ !\ \{R_2\} \qquad \cdot \vdash (f : \mathit{End}) : \circ \qquad \cdot \vdash R_1 : \mathrm{Row}_{\emptyset}{}^{\circ} \qquad R_1 \leqslant R_2}{\cdot; f : \mathit{End} \vdash \mathbf{do}\ \mathit{Choose}\ (); \mathbf{do}\ \mathit{Close}\ f : ()\ !\ \{R_2\}}$$

$R_1 = \{\mathit{Choose} : () \twoheadrightarrow^{\circ} \mathit{Bool}\}, R_2 = \{\mathit{Choose} : () \twoheadrightarrow^{\circ} \mathit{Bool}; \mathit{Close} : \mathit{End} \twoheadrightarrow^{\bullet} ()\}.$

More Precise Typing Rule for Sequencing

T-SeqSub

$$\frac{\Delta;\Gamma_1 \vdash M : A\,!\,\{R_1\} \qquad \Delta;\Gamma_2, x : A \vdash N : B\,!\,\{R_2\} \qquad \Delta \vdash (\Gamma_2, x : A) : Y \qquad \Delta \vdash R : \mathrm{Row}^{Y} \qquad R_1 \leqslant R_2}{\Delta;\Gamma_1 + \Gamma_2 \vdash \mathbf{let}\ x \leftarrow M\ \mathbf{in}\ N : B\,!\,\{R_2\}}$$

More Precise Typing Rule for Sequencing

$$\textsc{T-SeqSub}\quad \frac{\Delta; \Gamma_1 \vdash M : A\,!\,\{R_1\} \qquad \Delta; \Gamma_2, x : A \vdash N : B\,!\,\{R_2\} \qquad \Delta \vdash (\Gamma_2, x : A) : Y \qquad \Delta \vdash R : \mathrm{Row}^Y \qquad R_1 \leqslant R_2}{\Delta; \Gamma_1 + \Gamma_2 \vdash \mathbf{let}\ x \leftarrow M\ \mathbf{in}\ N : B\,!\,\{R_2\}}$$

Row subtyping relation $\boxed{R_1 \leqslant R_2}$

$$\frac{}{R \leqslant R} \qquad \frac{R_1 \leqslant R_2 \qquad R_2 \leqslant R_3}{R_1 \leqslant R_3} \qquad \frac{}{\cdot \leqslant \mu} \qquad \frac{R_1 \leqslant R_2}{\ell : \mathrm{Abs}; R_1 \leqslant \ell : P; R_2} \qquad \frac{R_1 \leqslant R_2}{\ell : P; R_1 \leqslant \ell : P; R_2}$$

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Although it is folklore that row polymorphism can replace row subtyping to some extent (especially for effect types), in settings like tracking control-flow linearity, a combination of them is better.

F°_{eff} Metatheory

Standard progress and preservation.

$\mathrm{F}^{\circ}_{\mathrm{eff}}$ Metatheory

Standard progress and preservation.

Lemma (Unlimited values are unlimited)

If $\Delta; \Gamma \vdash V : A$ *and* $\Delta \vdash A : \bullet$*, then* $\Delta \vdash \Gamma : \bullet$.

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Lemma (Unlimited operations are unlimited)

If $\Delta; \Gamma \vdash \mathcal{E}[(\mathbf{do}\ \ell\ V)^E] : A \,!\, \{\ell : A' \twoheadrightarrow^{\bullet} B', R\}$ *and* $\ell \notin \mathrm{bl}(\mathcal{E})$, *then there exists* $\Delta \vdash \Gamma = \Gamma_1 + \Gamma_2$ *s.t.* $\Delta \vdash \Gamma_1 : \bullet$ *and* $\Delta; \Gamma_1, y : B_\ell \vdash \mathcal{E}[\mathbf{return}\ y] : A \,!\, \{\ell : A' \twoheadrightarrow^{\bullet} B'; R\}$.

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By further defining a linearity-aware semantics, we can show that every linear value is used exactly once during evaluation.

Theorem (Evaluation linearity)

If M *is proper and* $M \overset{\mathcal{C}}{\underset{\mathcal{D}}{\leadsto}} N$, *then* N *is also proper and* $\mathscr{L}(M) \uplus \mathscr{L}(\mathcal{C}) = \mathscr{L}(N) \uplus \mathscr{L}(\mathcal{D})$.

What about Type Inference ?

Challenges of $\mathsf{F}^{\circ}_{\mathrm{eff}}$ type inference:

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Challenges of F°_{eff} type inference:

- First-class polymorphism $\Rightarrow$ prenex polymorphism
- Linear types and subkinding
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What about Type Inference ?

Challenges of F°_{eff} type inference:

- First-class polymorphism ⇒ prenex polymorphism
- Linear types and subkinding ⇒ qualified types (Quill[5])
- Row subtyping

[5] Morris, “The Best of Both Worlds: Linear Functional Programming without Compromise”, 2016.

What about Type Inference ?

Challenges of F°_{eff} type inference:

- First-class polymorphism $\Rightarrow$ prenex polymorphism
- Linear types and subkinding $\Rightarrow$ qualified types (QUILL[5])
- Row subtyping $\Rightarrow$ qualified types (ROSE[6])

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What about Type Inference ?

Challenges of F°_{eff} type inference:

- First-class polymorphism $\Rightarrow$ prenex polymorphism
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- Row subtyping $\Rightarrow$ qualified types (Rose[6])

Q°_{eff} : A ML-style variant of F°_{eff} based on qualified types with

- full type inference without any type annotations
- accurate tracking of control-flow linearity (even more accurate than F°_{eff})

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Future Work

- Shallow handlers:
  - linear shallow handlers can also introduce linear resources into continuations with a more complex behaviour than sequencing;
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  - Links supports first-class polymorphism using FreezeML[7];
  - non-trivial to extend FreezeML with qualified types.

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Future Work

- Shallow handlers:
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- Other classifications of effects:
  - besides linear ($\twoheadrightarrow^{\circ}$) and unlimited effects ($\twoheadrightarrow^{\bullet}$), our method can also be used for other classifications, like algebraic effects vs. higher-order effects.

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Thank you!

$F_{eff} + F^{\circ} = F^{\circ}_{eff}$

		Kinds K ::=
Value types	$A, B ::= \alpha \mid A \to^{Y} C \mid \forall^{Y} \alpha^{K}.C$	Type^{Y}
Computation types	$C, D ::= A\,!\,E$	Comp
Effect types	$E ::= \{R\}$	Effect
Row types	$R ::= \ell : P; R \mid \mu \mid \cdot$	$\mathsf{Row}^{Y}_{\mathcal{L}}$
Presence types	$P ::= \mathsf{Abs} \mid A \twoheadrightarrow^{Y} B \mid \theta$	$\mathsf{Presence}^{Y}$
Handler types	$F ::= C \rightrightarrows D$	Handler
Types	$T ::= A \mid R \mid P$	
Label sets	$\mathcal{L} ::= \emptyset \mid \{\ell\} \uplus \mathcal{L}$	
Linearity	$Y ::= \bullet \mid \circ$	
Type contexts	$\Gamma ::= \cdot \mid \Gamma, x : A$	
Kind contexts	$\Delta ::= \cdot \mid \Delta, \alpha : K$	
Values	$V, W ::= x \mid \lambda^{Y} x^{A}.M \mid \Lambda^{Y} \alpha^{K}.M$	
Computations	$M, N ::= V\,W \mid V\,T \mid (\mathbf{return}\ V)^{E} \mid (\mathbf{do}\ \ell\ V)^{E}$ $\mid \mathbf{let}\ x \leftarrow M\ \mathbf{in}\ N \mid \mathbf{handle}\ M\ \mathbf{with}\ H$	
Handlers	$H ::= \{\mathbf{return}\ x \mapsto M\} \mid \{\ell\ p\ r \mapsto M\} \uplus H$	

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Duality between value linearity and control-flow linearity

Instead, we can upcast the kind of row type.

For $M : A\,!\,\{(R : \mathsf{Row}_{\emptyset}{}^{Y})\}$, Y restricts the linearity of *its context*

- $Y = \circ$
  - operations in M are guaranteed to be handled linearly
  - M's continuation may contain linear resources
- $Y = \bullet$
  - no guarantee on the handling of operations in M
  - M's continuation must not contain linear resources

more restriction on its context

$$\frac{}{\vdash \bullet \leq \circ} \qquad \frac{\vdash Y \leq Y'}{\vdash \mathsf{Type}^{Y} \leq \mathsf{Type}^{Y'}} \qquad \frac{\vdash Y' \leq Y}{\vdash \mathsf{Presence}^{Y} \leq \mathsf{Presence}^{Y'}} \qquad \frac{\vdash Y' \leq Y}{\vdash \mathsf{Row}_{\mathcal{L}}{}^{Y} \leq \mathsf{Row}_{\mathcal{L}}{}^{Y'}}$$

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More Precise Typing Rule for Sequencing

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F°_{eff} Kinding Rules for Value Types

Kinding relation $\boxed{\Delta \vdash A : K}$

$$\frac{}{\vdash \bullet \leq \circ} \qquad \frac{\vdash Y \leq Y'}{\vdash \mathsf{Type}^{Y} \leq \mathsf{Type}^{Y'}}$$

$$\textsc{K-TyVar}\ \frac{}{\Delta, \alpha : K \vdash \alpha : K} \qquad \textsc{K-Forall}\ \frac{\Delta, \alpha : K \vdash C : \mathsf{Comp}}{\Delta \vdash \forall^{Y} \alpha^{K}. C : \mathsf{Type}^{Y}}$$

$$\textsc{K-Fun}\ \frac{\Delta \vdash A : \mathsf{Type}^{Y'} \quad \Delta \vdash C : \mathsf{Comp}}{\Delta \vdash A \to^{Y} B : \mathsf{Type}^{Y}} \qquad \textsc{K-Upcast}\ \frac{\Delta \vdash T : K \quad \vdash K \leq K'}{\Delta \vdash T : K'}$$

Extend to contexts $\boxed{\Delta \vdash \Gamma : Y}$

- $Y = \circ : \Gamma$ *may* contain linear variables (because of K-Upcast)
- $Y = \bullet : \Gamma$ only contains unlimited variables

Context splitting $\boxed{\Delta \vdash \Gamma = \Gamma_1 + \Gamma_2}$

- Variables with unlimited types appear in both Γ_1 and Γ_2
- Variables with linear types only appear in one of them

F°_{eff} Kinding Rules for Other Types

K-Effect
$$\frac{\Delta \vdash R : \mathsf{Row}^{Y}_{\emptyset}}{\Delta \vdash \{R\} : \mathsf{Effect}}$$

K-Comp
$$\frac{\Delta \vdash A : \mathsf{Type}^{Y} \qquad \Delta \vdash E : \mathsf{Effect}}{\Delta \vdash A \,!\, E : \mathsf{Comp}}$$

K-Handler
$$\frac{\Delta \vdash C : \mathsf{Comp} \qquad \Delta \vdash D : \mathsf{Comp}}{\Delta \vdash C \rightrightarrows D : \mathsf{Handler}}$$

K-Absent
$$\frac{}{\Delta \vdash \mathsf{Abs} : \mathsf{Presence}^{Y}}$$

K-Present
$$\frac{}{\Delta \vdash A \twoheadrightarrow^{Y} B : \mathsf{Presence}^{Y}}$$

K-EmptyRow
$$\frac{}{\Delta \vdash \cdot : \mathsf{Row}_{\mathcal{L}}{}^{Y}}$$

K-ExtendRow
$$\frac{\Delta \vdash P : \mathsf{Presence}^{Y} \qquad \Delta \vdash R : \mathsf{Row}_{\mathcal{L} \uplus \{\ell\}}{}^{Y}}{\Delta \vdash \ell : P; R : \mathsf{Row}_{\mathcal{L}}{}^{Y}}$$

F°_{eff} Metatheory (Cont.)

Definition (Properness)

A well-typed computation M or value V is proper if and only if,

1. for every sub-values W in it, if W *has* some type A which can be given kind $\mathsf{Type}^{\bullet}$, then $\mathscr{L}(W) = \emptyset$;
2. for every sub-computation N of form $\mathcal{E}[\mathbf{do}\ \ell\ V]$ where $\ell \notin \mathsf{bl}(\mathcal{E})$ in it, if N *has* some effect type $\{\ell : A_\ell \twoheadrightarrow^{\bullet} B_\ell; \ldots\}$, then $\mathscr{L}(\mathcal{E}) = \emptyset$.

Q°_{eff} Qualified Types

Row types	$R ::= \mu \mid \overline{\ell : A \twoheadrightarrow^{Y} B}$
Linearity	$Y ::= \phi \mid \bullet \mid \circ$
Types	$\tau ::= A \mid R \mid Y$
Predicates	$\pi ::= \underbrace{\tau_1 \leq \tau_2}_{\text{only compare linearity}} \mid \underbrace{R_1 \otimes R_2}_{\text{only compare label sets}} \mid R_1 \odot R_2 \sim R$
Qualified types	$\rho ::= A \mid \pi \Rightarrow \rho$
Type schemes	$\sigma ::= \rho \mid \forall \alpha.\sigma$

Back to the "print then close" example:

$$\begin{aligned}&\textbf{do}\ \textit{Print}\,\text{"42"}; \textbf{do}\ \textit{Close}\ f\ :\\ &\quad \forall \mu\, \phi_1\, \phi_2.((\textit{Print} : \phi_1) \otimes \mu, (\textit{Close} : \phi_2) \otimes \mu, \textit{File} \leq \phi_1) \Rightarrow ()\,!\,\{\mu\}\end{aligned}$$

As we know *File* is a linear type, we can further simplify it to:

$$\textbf{do}\ \textit{Print}\,\text{"42"}; \textbf{do}\ \textit{Close}\ f\ : \forall \mu\, \phi.((\textit{Print} : \circ) \otimes \mu, (\textit{Close} : \phi) \otimes \mu) \Rightarrow ()\,!\,\{\mu\}$$

Q°_{eff} Typing Rules

Typing relation $\boxed{P \mid \Gamma \vdash V : A}$ $\boxed{P \mid \Gamma \vdash M : C}$ $\boxed{P \mid \Gamma \vdash H : C \rightrightarrows D}$

Q-Abs

$$\frac{P \mid \Gamma, x : A \vdash M : C \qquad P \vdash \Gamma \leq Y}{P \mid \Gamma \vdash \lambda x.M : A \to^{Y} C}$$

"any type in Γ" $\leq Y$
$Y = \bullet$: all vars in Γ are unlimited
$Y = \circ$: essentially no restriction
$Y = \phi$: collect the constraint in P

Q-Handler

$$\frac{\begin{array}{c} H = \{\textbf{return}\ x \mapsto M\} \uplus \{\ell_i\ p_i\ r_i \mapsto N_i\}_i \\ D = B\,!\,\{R_2\} \qquad P \mid \Gamma, x : A \vdash M : D \\ [P \mid \Gamma, p_i : A_i, r_i : B_i \to^{Y_i} D \vdash N_i : D]_i \\ P \vdash \Gamma \leq \bullet \\ P \Rightarrow (\ell_i : A_i \twoheadrightarrow^{Y_i} B_i)_i \odot R \sim R_1 \qquad P \Rightarrow R \oslash R_2 \end{array}}{P \mid \Gamma \vdash H : A\,!\,\{R_1\} \rightrightarrows B\,!\,\{R_2\}}$$

$P \vdash \Gamma \leq \bullet$: all vars in Γ are unlimited
$P \Rightarrow (\ell_i : A_i \twoheadrightarrow^{Y_i} B_i)_i \odot R \sim R_1$: combination of $(\ell_i)_i$ and R
$P \Rightarrow R \oslash R_2$: R_2 contains R

Q-Seq

$$\frac{\begin{array}{c} P \mid \Gamma_1, \Gamma \vdash M : A\,!\,\{R_1\} \qquad P \mid \Gamma_2, \Gamma, x : A \vdash N : B\,!\,\{R_2\} \\ P \vdash \Gamma \leq \bullet \qquad P \Rightarrow R_1 \oslash R_2 \qquad P \vdash (\Gamma_2, x : A) \leq R_1 \end{array}}{P \mid \Gamma_1, \Gamma_2, \Gamma \vdash \textbf{let}\ x \leftarrow M\ \textbf{in}\ N : B\,!\,\{R_2\}}$$

$P \vdash \Gamma \leq \bullet$: all vars in Γ are unlimited
$P \Rightarrow R_1 \oslash R_2$: R_2 contains R_1
$P \vdash (\Gamma_2, x : A) \leq R_1$: "any type in $(\Gamma_2, x : A)$" $\leq$ "any label in R_1"
$R_1 = (\ell_i : Y_i)_i$: $[P \vdash (\Gamma_2, x : A) \leq Y_i]_i$
$R_1 = \mu$: collect the constraint in P

Q°_{eff} Type Inference

Almost standard Hindley-Milner type inference with qualified types.

Metatheory: Standard soundness and completeness.

Theorem (Soundness)

If $\theta;\Gamma \vdash V : A \dashv \theta', P, \Sigma$, *then* $\theta' P \mid \theta'(\Gamma|_{\Sigma}) \vdash V : \theta' A$. *The same applies to computation and handler typing.*

Theorem (Completeness)

If $P \mid \theta\Gamma \vdash V : A$, *then* $\iota;\Gamma \vdash V : A' \dashv \theta', Q, \Sigma$ *and there exists* θ'' *such that* $A = \theta''\theta' A'$, $P \Rightarrow \theta''\theta' Q$, *and* $\theta = (\theta''\theta')|_{\Gamma}$. *The same applies to computation and handler typing.*

Constraint solving? A seemingly correct graph algorithm for checking and simplifying constraints.

$\mathsf{Q}^{\circ}_{\mathsf{eff}}$ More Example

Consider the following function:

$$\lambda^{\bullet} f.\lambda^{\bullet} g.f\,();g\,()$$

The type inference of $\mathsf{F}^{\circ}_{\mathsf{eff}}$ infers the following principal type:

$$\begin{aligned}&\forall \alpha_1\,\alpha_2\,\mu_1\,\mu_2\,\phi_1\,\phi_2.(\phi_2 \leq \mu_1, \mu_1 \leqslant \mu_2)\\ &\quad\Rightarrow (() \to^{\phi_1} \alpha_1\,!\,\{\mu_1\}) \to^{\bullet} (() \to^{\phi_2} \alpha_2\,!\,\{\mu_2\}) \to^{\bullet} \alpha_2\,!\,\{\mu_2\}\end{aligned}$$

While in $\mathsf{F}^{\circ}_{\mathsf{eff}}$, the subtyping relation $\mu_1 \leqslant \mu_2$ requires $\mu_1 = \mu_2$, which is more restrictive.