# Effects, Linearity, and Modalities

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The University of Edinburgh

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# Effects and Handlers

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Typically ad hoc and hard-wired in programming languages.

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For a detailed introduction to the history of computational effects, see the first part of Nicolas Wu's keynote *Modular Higher-Order Effects* at PADL'24.

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Libraries in almost all mainstream languages even including C and C++.

Primitive supports in industrial languages (for both user-defined effects and low-level features):

- OCaml
- Unison
- WebAssembly (ongoing)
- Cangjie (ongoing)

# Algebraic Effects Specify Syntax

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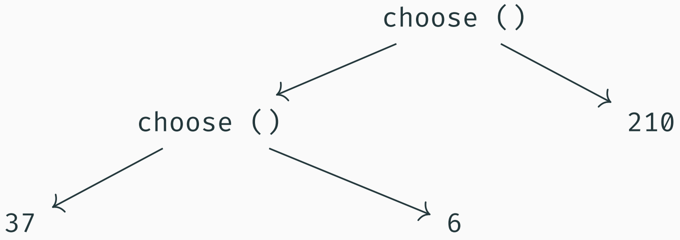

# Effect Handlers Provide Semantics

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all m = handle m () with
  return x   ⇒ [x]
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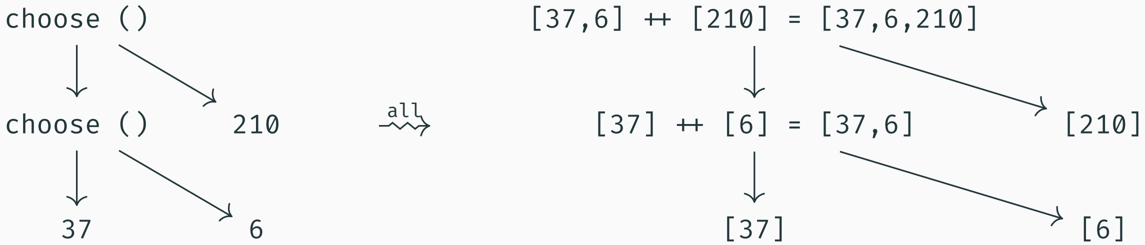

# Different Semantics without Changing Syntax

```
first m = handle m () with
  return x   ⇒ x
  choose _ r ⇒ r true
```

```
# first prog ⇝ 37
```

# Different Semantics without Changing Syntax

```
first m = handle m () with                    # first prog ⇝ 37
  return x   ⇒ x
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last m = handle m () with                     # last prog ⇝ 210
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# Composable Syntax and Semantics

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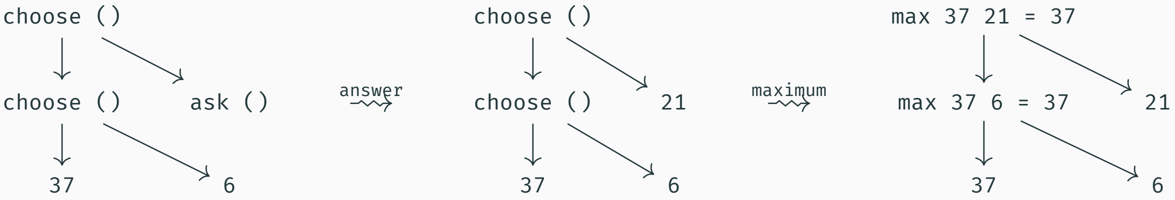


# composable semantics

## Different Points of View to Algebraic Effects and Handlers

- Modelling effects (ad hoc built-in effects, monads, monad transformers): *composable* and *customisable* effects interpretation in *direct style*.

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  *restorable* exception / try-catch with *continuations*.
- OOP programmers:
  (augmented) interfaces and implementations of objects
- Haskell programmers:
  freemonads and their algebras / folds / catamorphisms
- Coq programmers:
  (roughly) interaction trees

# Effect Systems

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prog : 1 --choose, ask--> Int
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all    : ∀ a . (1 --choose--> a) → List a  # handles choose
```

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$$\texttt{all} : \forall\, a\,.\,(1 \xrightarrow{\texttt{choose}} a) \to \texttt{List}\ a \quad \texttt{\# handles choose}$$

$$\texttt{answer} : \forall\, a\,.\,(1 \xrightarrow{\texttt{ask}} a) \to a \quad \texttt{\# handles ask}$$

# Effect Polymorphism

The types for `all` and `answer` are not composable!

$$\texttt{both : } \forall\ \texttt{a . (1} \xrightarrow{\texttt{ask, choose}} \texttt{a)} \to \texttt{a}$$

```
both m = all {answer m}    # {...} is short for (fun () → ...)
```

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The conventional solution is *effect polymorphism*, which introduces effect variables to quantify over other potential effects.

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$$\texttt{answer} : \forall\, \texttt{a e} .\ (\texttt{1} \xrightarrow{\texttt{ask, e}} \texttt{a}) \xrightarrow{\texttt{e}} \texttt{a}$$

Instantiating `e` with `choose` gives a compatible type

$$(\texttt{1} \xrightarrow{\texttt{ask, choose}} \texttt{a}) \xrightarrow{\texttt{choose}} \texttt{a}$$

# Effect Polymorphism

We also need to make other types effect polymorphic.

$$
\begin{array}{llll}
\texttt{pick} & : \forall\ \texttt{a}\ \texttt{e} & . & (\texttt{a},\ \texttt{a}) \xrightarrow{\texttt{choose},\ \texttt{e}} \texttt{a} \\
\texttt{prog} & : \forall\ \texttt{e} & . & 1 \xrightarrow{\texttt{choose},\ \texttt{ask},\ \texttt{e}} \texttt{Int} \\
\texttt{all} & : \forall\ \texttt{a}\ \texttt{e} & . & (1 \xrightarrow{\texttt{choose},\ \texttt{e}} \texttt{a}) \xrightarrow{\texttt{e}} \texttt{List a} \\
\texttt{answer} & : \forall\ \texttt{a}\ \texttt{e} & . & (1 \xrightarrow{\texttt{ask},\ \texttt{e}} \texttt{a}) \xrightarrow{\texttt{e}} \texttt{a} \\
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\end{array}
$$

Including existing “pure” functions like

$$\texttt{map} : \forall\ \texttt{a b e .}\ \texttt{(a} \xrightarrow{\texttt{e}} \texttt{b, List a)} \xrightarrow{\texttt{e}} \texttt{List b}$$

Works well with ML-style type inference via row polymorphism as in Koka and Links.

# More Examples: Generators

```
effect yield : Int ⇒ 1

                  yield, e        e
asList : ∀ e . (1 ────────→ 1) ──→ List Int
asList m = handle m () with
  return ()  ⇒ nil
  yield  x r ⇒ cons x (r ())

                        yield, e
gen : ∀ e . List Int ────────→ 1
gen xs = map (fun x → do yield x) xs; ()

> asList (gen [3,1,4,1,5,9])
[3,1,4,1,5,9]
```

# States

```
effect get : 1   ⇒ Int
effect put : Int ⇒ 1

state  : ∀ a . (1 --get, put, e--> a) -e-> Int -e-> (a, Int)
state m = handle m () with
  return x     ⇒ fun s → (x, s)
  get    () r  ⇒ fun s → r s s
  put    s' r  ⇒ fun s → r () s'
```

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                                  get, put, yield, e
prefixSum : ∀ e . List Int ─────────────────────→ 1
prefixSum xs = map (fun x → do put (do get + x); do yield (do get)) xs; ()
```

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prefixSum : ∀ e . List Int ───────────────→ 1
prefixSum xs = map (fun x → do put (do get + x); do yield (do get)) xs; ()

> asList (fun () →
    state (fun () → prefixSum [3,1,4,1,5,9]) 0; ())
[3,4,8,9,14,23]
```

# Linearity and Control-Flow Linearity

# Linear Resources

*Some of the best things in life are free; and some are not.* (*Philip Wadler, A taste of linear logic, 1993*)

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Linear resources must be used exactly once.

- File handles.
- (Session-typed) communication channels.
- Network connections.
- Memory management (affine).

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❎ Type error: f has a linear type File but is used twice.
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writer'' : File → String → 1
writer'' f s = let f' = write f s in close f'
❎ Type error: f has a linear type File but is captured in
   a non-linear function of type String → 1.
```

# (Multi-Shot) Effect Handlers Break Linear Types

$$\texttt{dubiousWriter} : \texttt{File} \xrightarrow{\texttt{Choose}} \texttt{1}$$

```
dubiousWriter f = let f' = write f (pick "6" "37") in close f'
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# (Multi-Shot) Effect Handlers Break Linear Types

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> all (fun () → dubiousWriter (open "file.txt"))
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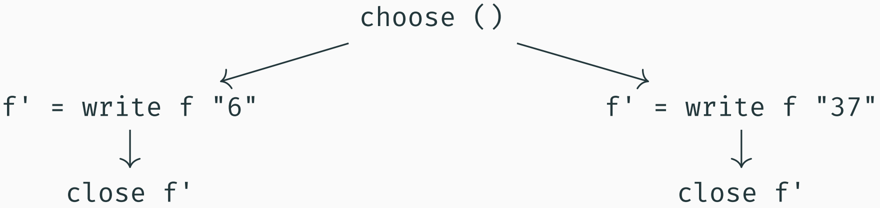

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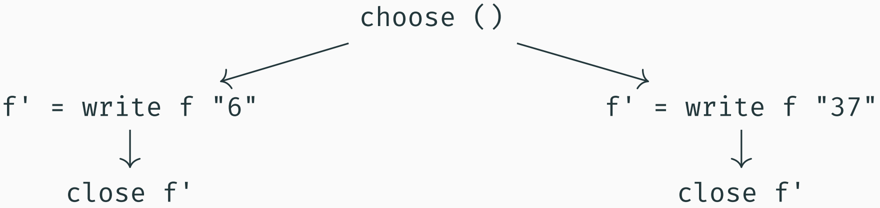


Conventional linear type systems only track *value linearity*; they assume continuations are used linearly. However, effect handlers enable more flexible uses of continuations.

# Tracking Control-Flow Linearity

Classify operations into two categories:

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> first (fun () → dubiousWriter (open "file.txt"))
[()]  # "6" is written
```

# More in the Paper

I have omitted all details.

Two calculi which track control-flow linearity in the paper:

- $F^{\circ}_{\text{eff}}$ : A system F-style core calculus with subkinding-based linear types and row-based effect types. Requires syntactic overheads.
- $Q^{\circ}_{\text{eff}}$ : An ML-style calculus with linear and effect types both based on qualified types. Infers principal types with no extra annotation.

paper

blog post

# Modal Effect Types

# Verbosity of Conventional Effect Types

Effect polymorphism requires annotating almost all function arrows with effect variables.

$$\texttt{gen} \quad : \forall\ \texttt{e}\ .\ \texttt{List Int} \xrightarrow{\texttt{yield, e}} \texttt{1}$$

$$\texttt{asList} : \forall\ \texttt{e}\ .\ (\texttt{1} \xrightarrow{\texttt{yield, e}} \texttt{1}) \xrightarrow{\texttt{e}} \texttt{List Int}$$

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Even for innocent higher-order functions which do not use or handle effects at all.

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This verbosity severely hinders the adoption of effect systems in industrial languages: signatures of much existing library code must be rewritten no matter whether they use effects or not.

## Invisible Effect Polymorphism

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➡️ omit effect variables when they are the same one.

$$
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- Non-trivial mental programming model — programmers still need to understand effect polymorphism and reason about code by mentally desugaring.
- Broken syntactic abstraction — explicit effect variables may still appear in error messages and intermediate information provided by language server protocols.

A syntactical abstraction is neither satisfying from a theoretical point of view — is there a more fundamental system that captures the intuition of invisible effect polymorphism?

# Effect Contexts

*Managing effectful computations is about managing open terms. (Leo White)*

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HOAS: use bindings of the meta-lang to encode both variable and effect bindings.

# Being in Scope ❎➡️ Being Used

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For the typing judgement of `main`, $E$ is empty.

# Absolute Modalities

How do programmers specify that `main` is pure?

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All global programs are implicitly boxed with `[]`.

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Similar to contextual modal types.

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How to give types to handlers?

$$\texttt{asList} : \forall\, \texttt{e} \,.\, (\texttt{1} \xrightarrow{\texttt{yield},\ \texttt{e}} \texttt{1}) \xrightarrow{\texttt{e}} \texttt{List Int}$$

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asList m = handle m () with
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Using absolute modalities to specify their differences would be too verbose:

```
asList : ∀ e . [e]([yield, e](1 → 1) → List Int)
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# Relative Modalities

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Relative modalities have the general form `<L|D>` where `L` and `D` are effects.

# Escaping Handlers

What type should we give to `answer`?

```
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  ask () r ⇒ r 21
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We cannot give type `∀ a . <ask>(1 → a) → a` to it!

```
foo : 1 → Int
foo = answer (fun _ → do ask)  # ask escapes from handler scope
> foo ()
❎ Runtime error: ask is used but not handled
```

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> foo ()
❎ Runtime error: ask is used but not handled
```

Instead, the typing rule always wraps the return type of handlers with extension modalities of the operations they handle.

```
answer : ∀ a . <ask>(1 → a) → <ask>a
```

# Absolute Kinds

Recall that we directly have

```
asList : <yield>(1 → 1) → List Int
```

instead of

```
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This is sound because the unit type cannot carry any escaped operation.

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In general we introduce a kind system with subkinding `Abs ⩽ Any` where

- values of type `A : Abs` do not rely on the ambient effect context, and
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1 : Abs      Int : Abs      [yield](1 → 1) : Abs      (Bool, String) : Abs
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```
1 : Abs      Int : Abs      [yield](1 → 1) : Abs      (Bool, String) : Abs
```

Generalise to polymorphic types naturally

```
answer : ∀ [a] . <ask>(1 → a) → a  # short for ∀ a : Abs
```

# Modular Effectful Programming with Modal Effect Types

```
> asList <yield>(fun () → gen [3,1,4,1,5,9])
# [3,1,4,1,5,9] : List Int

> asList <yield>(fun () →
    state <get,put>(fun () → gen' [3,1,4,1,5,9]) 0; ())
# [3,4,8,9,14,23] : List Int
```

Unfortunately, my type inference cannot infer all modality introduction 🥲.

# Comparing with the Syntactic Sugar

For most common types they give similar results.

$$1 \xrightarrow{\texttt{choose, ask}} \texttt{Int}$$

$$\forall\ a\ .\ (1 \xrightarrow{\texttt{choose}} a) \to \texttt{List}\ a$$

$$\forall\ a\ .\ (1 \xrightarrow{\texttt{ask}} a) \to a$$

$$\forall\ a\ b\ .\ (a \to b,\ \texttt{List}\ a) \to \texttt{List}\ b$$

```
[choose, ask](1 → Int)
∀ [a] . <choose>(1 → a) → List a
∀ [a] . <ask>(1 → a) → a
∀ a b . (a → b, List a) → List b
```

# Comparing with the Syntactic Sugar

For most common types they give similar results.

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| $1 \xrightarrow{\texttt{choose, ask}} \texttt{Int}$ | $\texttt{[choose, ask](1} \to \texttt{Int)}$ |
| $\forall\ a\ .\ (1 \xrightarrow{\texttt{choose}} a) \to \texttt{List}\ a$ | $\forall\ [a]\ .\ \langle\texttt{choose}\rangle(1 \to a) \to \texttt{List}\ a$ |
| $\forall\ a\ .\ (1 \xrightarrow{\texttt{ask}} a) \to a$ | $\forall\ [a]\ .\ \langle\texttt{ask}\rangle(1 \to a) \to a$ |
| $\forall\ a\ b\ .\ (a \to b,\ \texttt{List}\ a) \to \texttt{List}\ b$ | $\forall\ a\ b\ .\ (a \to b,\ \texttt{List}\ a) \to \texttt{List}\ b$ |

The contextual reading could give better types in some cases.

| | |
|---|---|
| $1 \xrightarrow{\texttt{choose, ask}} 1 \xrightarrow{\texttt{choose, ask}} 1$ | $\texttt{[choose, ask](1} \to 1 \to 1)$ |
| $1 \xrightarrow{\texttt{choose}} 1 \xrightarrow{\texttt{choose, ask}} 1$ | $\texttt{[choose](1} \to \langle\texttt{ask}\rangle(1 \to 1))$ |
| $\forall\ f\ .\ (\forall\ e\ .\ (1 \xrightarrow{\texttt{ask, e}} 1) \xrightarrow{e} 1) \xrightarrow{f} 1$ | $[\,](\langle\texttt{ask}\rangle(1 \to 1) \to 1) \to 1$ |

Check out our preprint for a formal compositional encoding from left to right.

# One More Example: Re-generating

Process all generated numbers with a function.

```
regen : [yield]((Int → Int) → <yield>(1 → 1) → 1)
regen f m = handle m () with
  return ()  ⇒ ()
  yield  s r ⇒ do yield (f s); r ()
```

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In contrast, conventional effect systems (e.g., the one used in Koka) usually give

$$\texttt{regen} : \forall\ e\ .\ (\texttt{Int} \xrightarrow{\texttt{yield},\ e} \texttt{Int}) \xrightarrow{e} (\texttt{1} \xrightarrow{\texttt{yield},\ \texttt{yield},\ e} \texttt{1}) \xrightarrow{\texttt{yield},\ e} \texttt{1}$$

# Cooperative Concurrency

```
data Proc = proc (List Proc → ())

push : ∀ a . a → List a → List a
push x xs = xs ++ cons x nil
```

```
next : List Proc → ()
next q = case q of
  nil              → ()
  cons (proc p) ps → p ps
```

```
schedule : <ufork, suspend>(1 → 1) → List Proc → 1
schedule m = handle m () with
  return  ()   ⇒ fun q → next q
  suspend () r ⇒ fun q → next (push (proc (r ())) q)
  ufork   () r ⇒ fun q → r true (push (proc (r false)) q)
```

# Theoretical Foundation: (Simply) Multimodal Type Theory

Modal effect types have a solid theoretical foundation based on (the simply-typed fragment) of multimodal type theory, a dependent type theory parameterised by a mode theory, which specifies the structure of modes, modalities, and their transformations.

# More in the Paper

**MET:** A core calculus following simple *multimodal type theory*.

*Encoding* a fragment of conventional effect types into MET

**METE:** Extension with *effect variables*.

**METEL:** A surface language with *sound and complete type inference*.